Scientific Computing 1

Winter term 2017/18
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Sheet 7

Submission on **Thursday, 7.12.17**.

Exercise 1. (scaling)

Let $K \subset \mathbb{R}^n$ be an open and bounded set and $u \in W_p^k(K)$ for some $1 \leq p < \infty$. Furthermore, let $0 < d \in \mathbb{R}$ and $L(x) = dx$ be a scaling operation on $\mathbb{R}^n$ and define $\hat{K} = L(K) = \{L(x) \mid x \in K\}$ as well as new coordinates $\hat{x} = L(x)$. For the function $\hat{u} \in W_p^k(\hat{K})$ defined via $\hat{u}(\hat{x}) = u(x)$, show that

$$|\hat{u}|_{W_p^k(\hat{K})} = d^{n/p-k} \, |u|_{W_p^k(K)} \, .$$

(4 points)

Exercise 2. (integral kernel operator)

Let $U, V \subset \mathbb{R}^n$ be open and bounded, $f\colon V \to \mathbb{R}$ measurable and $K\colon \bar{U} \times \bar{V} \to \mathbb{R}$ continuous. For $x \in U$ define $(T_K f)(x) \coloneqq \int_V K(x,y) f(y) \, \mathrm{d}y$. Show that

a) T_K is a linear and bounded operator from $\mathcal{C}(\bar{V})$ to $\mathcal{C}(\bar{U})$,

b) $\|T_K\|_{\mathcal{L}(\mathcal{C}(\bar{V}),\mathcal{C}(\bar{U}))} = \sup_{x\in\bar{U}} \int_V |K(x,y)| \, \mathrm{d}y \, .$

(4 points)