Scientific Computing 1

Winter term 2017/18
Priv.-Doz. Dr. Christian Rieger
Christopher Kacwin

universitätbonn

Sheet 9

Submission on **Thursday, 21.12.17**.

Exercise 1. (block matrix)

Consider the block matrix

$$C = \begin{bmatrix} A & B \\ B^\top & 0 \end{bmatrix} \in \mathbb{R}^{(m+n)\times(m+n)}$$

with $A \in \mathbb{R}^{m\times m}$ and $B \in \mathbb{R}^{m\times n}$. Assume that $\operatorname{rank}(B) = n$ and that A is positive definite on $\ker(B^\top)$. Show that C is invertible.

(4 points)

Exercise 2. (Neumann problem)

Let $\Omega \subset \mathbb{R}^n$ be an open, bounded and convex domain with smooth boundary $\partial\Omega$ and normal unit vector $n \colon \partial\Omega \to \mathbb{R}^n$. Consider the free Neumann Problem

$$\begin{aligned} -\Delta u &= f \quad \text{in} \quad \Omega\,, \\ \partial_n u &= g \quad \text{on} \quad \partial\Omega\,. \end{aligned}$$

a) Assume that a strong solution exists. Show that

$$\int_\Omega f \,\mathrm{d}x + \int_{\partial\Omega} g \,\mathrm{d}S = 0\,.$$

b) If u is a weak solution, then u plus a constant is also a weak solution, therefore we may enforce the additional constraint

$$\int_\Omega u \,\mathrm{d}x = 0\,.$$

Show that the bilinear form $a(u,v) = \int_\Omega \nabla u \nabla v \,\mathrm{d}x$ on $H^1(\Omega)$ is elliptic (w.r.t the $H^1(\Omega)$-norm) on the set of functions $v \in H^1(\Omega)$ that satisfy the constraint.

(4 points)