Scientific Computing I

Wintersemester 2018/2019
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Exercise Sheet 10.

Due date: **Tue, 8.1.2019**.

Exercise 1. (6 Points)

Let $\mathcal{T}_h$ a uniform triangulation of Ω and suppose that S_h belongs to an affine family of finite elements. Further, suppose the nodes of a nodal basis of S_h are $z_1, \ldots, z_N$ with $N = N_h = \dim S_h$. Verify that for some constant c independent of h, the following inequality holds:

$$c^{-1}\|v\|_{0,\Omega}^2 \leq h^2 \sum_{i=1}^{N} |v(z_i)|^2 \leq c\|v\|_{0,\Omega}^2 \quad \text{for all } v \in S_h. \tag{1}$$

Exercise 2. (6 Points)

Prove the Bramble-Hilbert lemma for $t = 1$ by choosing the interpolator Iv to be the constant function

$$Iv := \frac{\int_\Omega v \,\mathrm{d}x}{\int_\Omega \,\mathrm{d}x}. \tag{2}$$

Exercise 3. (6 Points)

Let Ω be a polygonal two-dimensional domain. Consider a family of triangle-based quasi-uniform triangulations $\{\mathcal{T}_h\}_h$ of Ω and denote the space of linear finite elements by V_h. Show that

$$\|v_h\|_\infty \leq Ch^{-1}\|v_h\|_0 \quad \text{for all } v_h \in V_h \tag{3}$$

for a constant C independent of h.

Exercise 4. (6 Points)

With the same definitions as in Exercise 2, show that the discrete solution u_h of the Poisson equation with homogeneous boundary conditions satisfies

$$\|u - u_h\|_\infty \leq Ch\|u\|_2. \tag{4}$$

Hint: Use (3).