Scientific Computing I

Wintersemester 2018/2019
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Exercise Sheet 11.

Due date: **Tue, 15.1.2019**.

The points you may obtain from this problem sheet are bonus points

Exercise 1. (Preconditioning) (3+3 Points)

A matrix $A \in \mathbb{R}^{n\times n}$ is called row equilibrated if $\sum_{k=1}^{n} |a_{jk}| = 1$ for all $j = 1, \dots, n$. Prove the following properties.

a) Let $A \in \mathbb{R}^{n\times n}$ be a row equilibrated and regular matrix. Then, $\operatorname{cond}_\infty(A) \le \operatorname{cond}_\infty(DA)$, for each regular and diagonal matrix $D \in \mathbb{R}^{n\times n}$.

b) Let $A \in \mathbb{R}^{n\times n}$ be a regular matrix. Define $T := \operatorname{diag}\{\alpha_1^{-1}, \dots, \alpha_n^{-1}\}$ where $\alpha_j = \sum_{k=1}^{n} |a_{jk}|$. Then $\operatorname{cond}_\infty(TA) \le \operatorname{cond}_\infty(A)$.

Exercise 2. (Condition number) (6 Points)

Show that the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \tag{1}$$

is positive definite and that its condition number is 4.
Hint: Consider the quadratic form associated with A.

Exercise 3. (6 Points)

Let V denote a Hilbert space with inner product $a(\cdot,\cdot)$ and U a closed subspace of V. For $g \in V$ define $U_g = \{v + g \mid v \in U\}$. Prove that the following statements are equivalent,

a) $u \in U_g$ satisfies $a(u, v) = 0$ for all $v \in U_0$,

b) u minimizes $a(v, v)$ over $v \in U_g$.

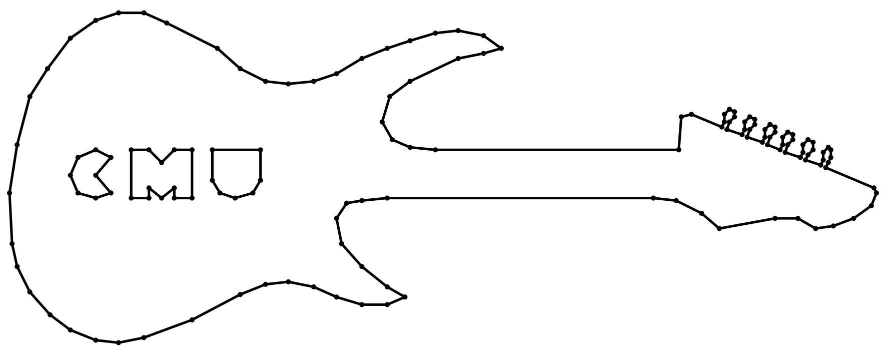

Figure 1: Geometry for the programming task.

Programming Exercise 1. (Optional) (1+9 Points)

We want to solve the Poisson problem

$$\begin{aligned} -\Delta u &= 1 \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned} \tag{2}$$

using the finite element method. Ω is the geometry depicted in Figure **??**. The corresponding `.poly` file describing it can be obtained at `https://www.cs.cmu.edu/~quake/guitar.poly`.

Definition 1. *Given a matrix* $M \in \mathbb{R}^{n\times n}$*, its lumped approximation is given as* $\operatorname{diag}\{M(1,\ldots,1)^\top\}$.

a) Use `triangle` to produce a triangulation of Ω and the corresponding `.ele` and `.node` files.

b) Modify the CG method from the previous problem sheets such that it allows the usage of a preconditioner. Approximate the solution of the system of equations arising after discretization of (**??**) with P_1 linear elements for the above described Ω. Use the lumped mass matrix approximation as preconditioner and compare with the non-preconditioned version.

Hint: With the PETSc CG implementation the preconditioner P can be easily realized by coding the action of it to a given vector ($v \to Pv$) and passing it as argument to the function `PCSetType(PC pc, PCType)`. See for example `https://www.mcs.anl.gov/petsc/petsc-3.10/docs/manualpages/PC/PCSHELL.html#PCSHELL`.

The programming task can be solved in groups of at most two students. Please present your solutions during the week of January 21–25, 2019 in either Mathematics CIP-Pool. Please make sure to pick a slot ahead of time by signing into the corresponding CIP-Pool list for this lecture.