Scientific Computing I

Wintersemester 2018/2019

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Exercise Sheet 12.

Due date: **Tue, 22.1.2019**.

The points you may obtain from this problem sheet are bonus points

Exercise 1. (6 Points)

Let A and B be M-matrices with $B \geq A$ (component wise). Prove that $0 \leq B^{-1} \leq A^{-1}$ and $\|B^{-1}\|_\infty \leq \|A^{-1}\|_\infty$.

Exercise 2. (6 Points)

Compute the incomplete LU factorization with zero fill-in, ILU(0) of the following matrix

$$\begin{pmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix}. \tag{1}$$

Exercise 3. (6 Points)

Let

$$u(x) := \begin{cases} \frac{1}{2}x^2 + x + 1 & \text{for } x < 0, \\ -\frac{1}{2}x^2 + x + 1 & \text{for } x \geq 0. \end{cases} \tag{2}$$

Does u have a second order derivative in the strong or the weak sense? Compute it according to your answer.

Exercise 4. (2+4 Points)

Let $\Omega \in \mathbb{R}^d$ be a bounded Lipschitz domain, $f \in L^2(\Omega)$ and $g \in L^2(\partial\Omega)$. For $\beta \geq 0$ and $\nu \in \mathbb{R}^d$ consider the Poisson problem

$$-\Delta u = f \quad \text{in } \Omega, \tag{3a}$$

$$\beta u + \nu^\top \nabla u = g \quad \text{on } \partial\Omega. \tag{3b}$$

a) Let $V := H^1(\Omega)/\mathbb{R}$, that is, V consists of classes of $H^1(\Omega)$ functions differing only by an additive constant. Prove that V with $(u, v)_V := (\nabla u, \nabla v)_0$ is a Hilbert space and that in the case $\beta = 0$ the problem (3) as a unique solution in V.

b) If $\beta > 0$, find a suitable Hilbert space V and a bilinear form $a(\cdot, \cdot) : V \times V \to \mathbb{R}$ such that (3) has a unique solution when posed in this space.