Numerical Algorithms

Winter term 2019/20
Prof. Dr. Carsten Burstedde
Christopher Kacwin

UNIVERSITÄT BONN

Sheet 10

Submission on **Tuesday, 7.1.20 in class**.

Exercise 1. (CFL-number)

Let $\Omega \subset \mathbb{R}^2$ be a smooth and bounded domain. Consider the parabolic PDE

$$\partial_t u = a\Delta u \qquad \text{in } \Omega \times [0,T] \tag{10.1}$$
$$u = 0 \qquad \text{on } \partial\Omega \times [0,T] \tag{10.2}$$
$$u(\cdot,0) = u_0 \qquad \text{on } \Omega \tag{10.3}$$

with $a > 0$. If this PDE is discretized using the implicit Euler scheme in time and continuous piecewise linear elements in space, how does the combined error bound look like? How should the stepsize Δt and mesh refinement parameter h be chosen to obtain the optimal rate?

(5 points)

Programming Exercise 1. (Solving a PDE over a curved domain, part III)

We will extend the solution of an elliptic PDE that we have worked on so far to parabolic PDEs. The circular domain, mesh, and ansatz functions are as before.
First of all, invent a C_2 solution for a parabolic PDE on the circle and explicitly derive initial and boundary conditions. Choose the final time T.
We have a new parameter, the time step size Δt. Implement the Crank-Nicholson method to solve the PDE to the final time. In each time step, there will be an elliptic system to be solved.

1. Verify that the method works correctly for $\Delta t = T = 0$ (one time step).
2. Compute the L_2 norm over Ω of the solution at T. Use proper quadrature to make sure that the integration error is smaller than the discretization errors in space and time.
3. Plot how the error depends on Δt when $h = 1/M$ is fixed. What is the order of convergence and what is the limit?
4. Plot how the error depends on h when Δt is fixed. What is the order of convergence this time and what is the limit?

(20 points)

The programming exercise has to be presented during the exercise class on Wednesday, January 22, 2020 in the computer room 2.038. You may bring your own laptop to class or prepare a solution that can be executed on the INS computers.