Sheet 11

Submission on Tuesday, 14.1.20 in class.

Exercise 1. (residual error)
Consider Exercise 1 from Sheet 7. Derive a similar identity for the weak formulation of the second order elliptic PDE
\[- \text{div}(A \nabla u(x)) + cu(x) = f(x) \text{ on } \Omega\] (11.1)
with homogeneous Dirichlet boundary conditions, \(A \in \mathbb{R}^{2 \times 2}\) is symmetric positive definite and \(c > 0\).

(5 points)

Exercise 2. (norm equivalence)
We prepare the derivation of an \(H_1\)-error estimator for the situation in Exercise 1. Show that there is a constant \(C = C(A, c, \Omega) > 0\) s.t.
\[\|u - u_h\|_1 \leq C \sup_{v \in H_0^1(\Omega)} \frac{1}{\|v\|_1} \int_{\Omega} [\nabla(u - u_h)A \nabla v + c(u - u_h)v] \, dx.\] (11.2)

(5 points)

Exercise 3. (Gauss-Lobatto points)
Consider the interval \(I = [-1, 1]\) with Gauss-Lobatto points \(N = \{-1, -\sqrt{5}^{-1}, \sqrt{5}^{-1}, 1\}\).

a) Construct a nodal basis w.r.t. \(N\) that spans the polynomials on \(I\) up to order 3.

b) Consider the Gauss Lobatto points on \(I_1 = [-1, 0]\) and \(I_2 = [0, 1]\). Calculate the matrices \(P^r\) (c.f equation (2.9.4) from the lecture).

(5 points)

Exercise 4. (triangle bisection)
For a triangle \(T\) with smallest angle \(\tau\) bisect the triangle along its longest edge and do the same for all subtriangles arbitrarily often. Prove that all emerging triangles have smallest angle at least \(\frac{\tau}{2}\).

(5 points)