Numerical Algorithms

Winter term 2019/20
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Sheet 11

Submission on **Tuesday, 14.1.20 in class**.

Exercise 1. (residual error)

Consider Exercise 1 from Sheet 7. Derive a similar identity for the weak formulation of the second order elliptic PDE

$$-\operatorname{div}(A\nabla u(x)) + cu(x) = f(x) \text{ on } \Omega \tag{11.1}$$

with homogeneous Dirichlet boundary conditions, $A \in \mathbb{R}^{2\times 2}$ is symmetric positive definite and $c > 0$.

(5 points)

Exercise 2. (norm equivalence)

We prepare the derivation of an H_1-error estimator for the situation in Exercise 1. Show that there is a constant $C = C(A, c, \Omega) > 0$ s.t.

$$\|u - u_h\|_1 \leq C \sup_{v \in H_0^1(\Omega)} \frac{1}{\|v\|_1} \int_\Omega [\nabla(u - u_h)A\nabla v + c(u - u_h)v] \, \mathrm{d}x\,. \tag{11.2}$$

(5 points)

Exercise 3. (Gauss-Lobatto points)

Consider the interval $I = [-1, 1]$ with Gauss-Lobatto points $N = \left\{-1, -\sqrt{5}^{-1}, \sqrt{5}^{-1}, 1\right\}$.

a) Construct a nodal basis w.r.t. N that spans the polynomials on I up to order 3.

b) Consider the Gauss Lobatto points on $I_1 = [-1, 0]$ and $I_2 = [0, 1]$. Calculate the matrices P^r (c.f equation (2.9.4) from the lecture).

(5 points)

Exercise 4. (triangle bisection)

For a triangle T with smallest angle τ bisect the triangle along its longest edge and do the same for all subtriangles arbitrarily often. Prove that all emerging triangles have smallest angle at least $\frac{\tau}{2}$.

(5 points)