Numerical Algorithms

Winter term 2019/20
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Sheet 12

Submission on **Tuesday, 21.1.20 in class**.

Exercise 1. (Schur complement)

Let

$$M = \begin{pmatrix} A & B \\ B^\top & C \end{pmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}, \tag{12.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times m}$ are symmetric positive definite. The Schur decomposition is given by

$$M = LU = \begin{pmatrix} I & 0 \\ B^\top A^{-1} & I \end{pmatrix} \begin{pmatrix} A & B \\ 0 & S \end{pmatrix}, \tag{12.2}$$

with $S \in \mathbb{R}^{m\times m}$ called Schur complement of A in M.

a) Compute S in terms of A, B and C.

b) Show that S is positive definite if and only if M is positive definite.

Let $P \in \mathbb{R}^{m\times m}$ be symmetric positive definite, and consider

$$Q = L \begin{pmatrix} A & B \\ 0 & P \end{pmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}. \tag{12.3}$$

c) Show that an eigenvalue $\lambda \neq 1$ of $P^{-1}S$ is also an eigenvalue of $Q^{-1}M$. Interpret this in the context of preconditioners.

(0 points)

Exercise 2. (error estimator)

Consider the setting in Exercise 1 and 2 from Sheet 11. Derive an error estimator for this problem together with an estimate of the global H_1-error, using appropiate residual terms R_T and R_h.

(0 points)

Exercise 3. (Alternative successor algorithm)

a) Write down the function `sibling` (x, l, j) that creates the j-th child of the parent of the element (x, l), $l > 0$, based on bitwise operations.

b) Write an alternative, recursive `successor` function that uses `childid`, `sibling`, `parent`, and `child`.

(0 points)