



Numerical Algorithms

Winter term 2019/20
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Sheet 8

Submission on **Tuesday, 10.12.19** in class.

Exercise 1. (preconditioning)

For a linear system of equations $Ax = b$ with $A \in \mathbb{R}^{n \times n}$ symmetric and positive definite, consider a preconditioner $C \in \mathbb{R}^{n \times n}$ which is also symmetric and positive definite. Show that for a certain choice of a scalar product, the preconditioned CG-method can be stated as the standard CG-method applied to the system $C^{-1}Ax = C^{-1}b$.

(5 points)

Exercise 2. (the CG method and dual spaces)

Write down the conjugate gradient method in terms of the coefficient vectors X_j , P_j , R_j , and Z_j as derived in the lecture. Propose a stopping criterion that is relative with respect to the initial residual. Compare with the PCG method found in standard literature.

Furthermore, prove by induction the usual properties:

1. A_0 -conjugacy of the search directions P_j ,
2. M_0 -conjugacy of the (preconditioned residual) vectors R_j .

(5 points)