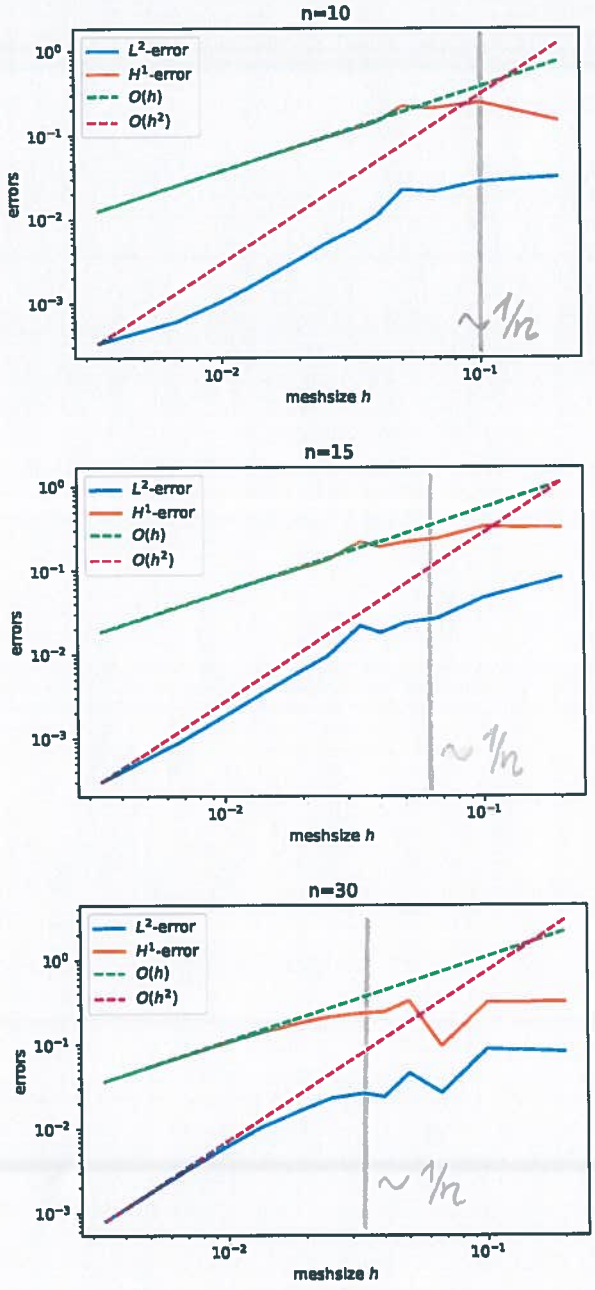
Programming exercise 10.1a): Oscillating coefficient in 1D

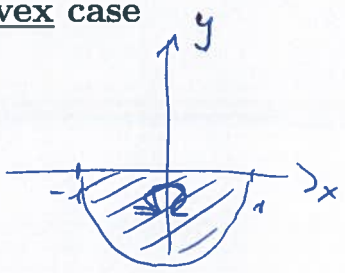


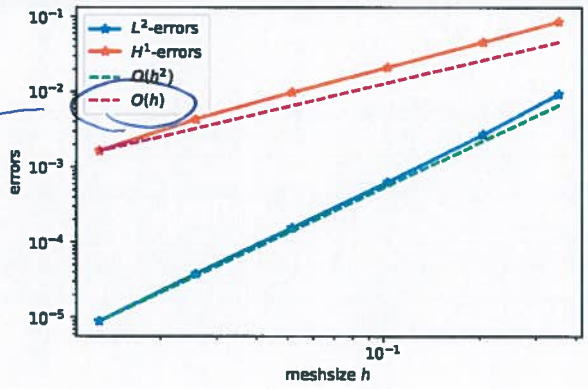
- convergence rates ($O(h)$ w.r.t. H^1, $O(h^2)$ w.r.t. L^2) as expected from the lecture due to H^2-regularity of the problem
- Due to scaling of the H^2-norm of the solution w.r.t. n the characteristic behaviour starts only for $h \ll 1/n$ (cf. exercise 8.2)

Programming exercise 10.1b): "Reentrent corner" - convex case

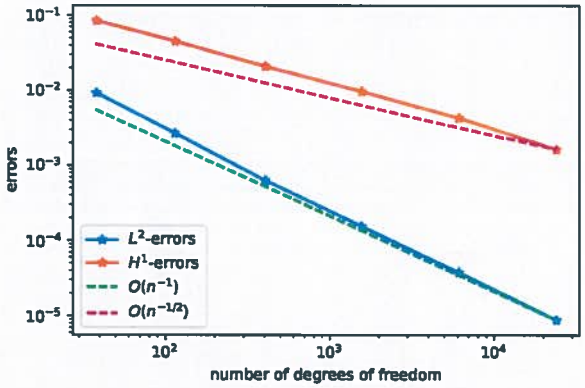
Piecewise linear FEM

Domain:



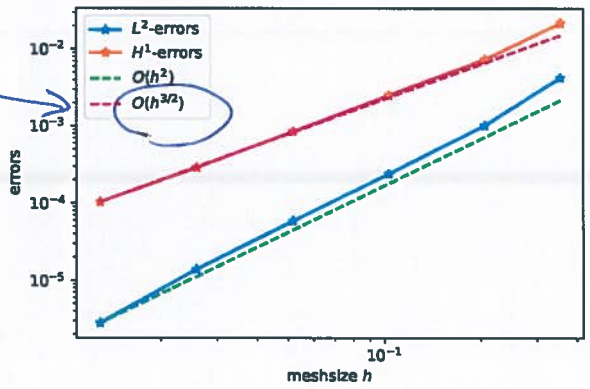


H^2-regular problem:
(smooth coefficient, convex domain + L^2-right hand side)

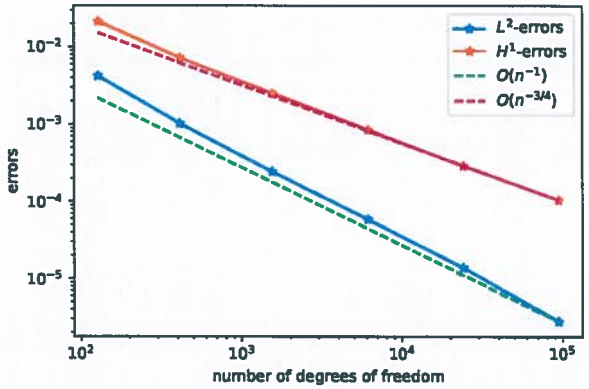


↪ convergence rates as expected

Piecewise quadratic FEM



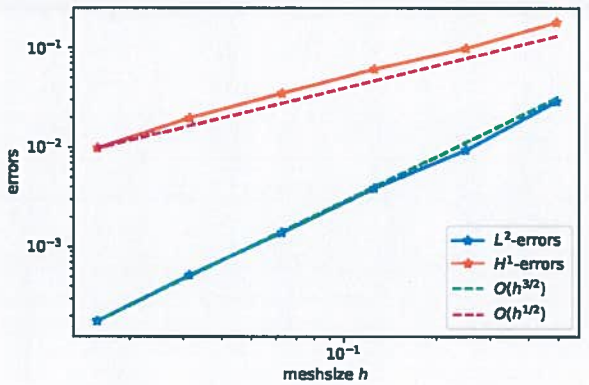
Pcw. Quadratic FEM are able to exploit the high regularity of the solution and produce a better approx. w.r.t. H^1-norm than pcw. linear FEM.

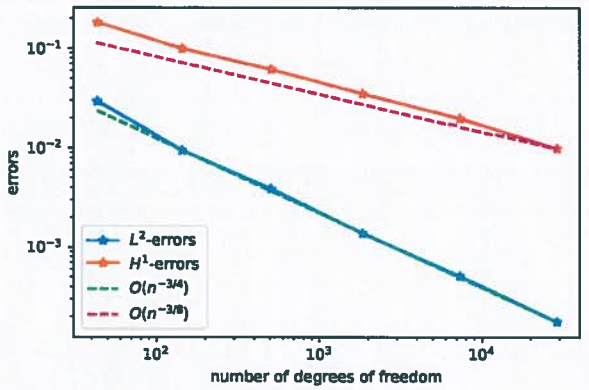


[Comment: The rates w.r.t. Ndof get worse in higher dimensions!]

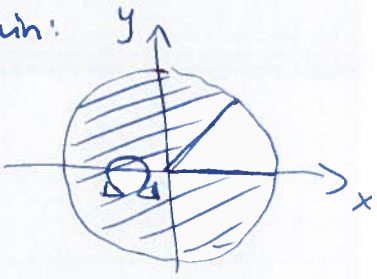
Programming exercise 10.1b): Reentrent corner - nonconvex case

Piecewise linear FEM





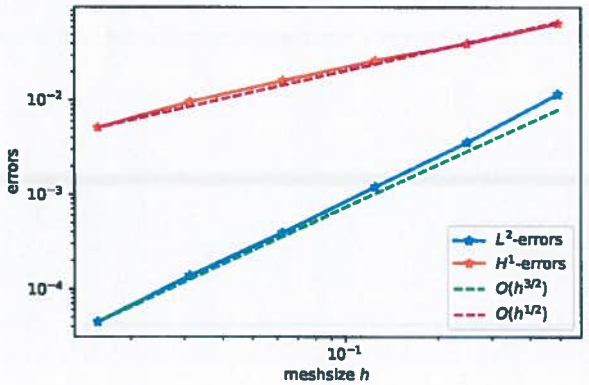
Domain:

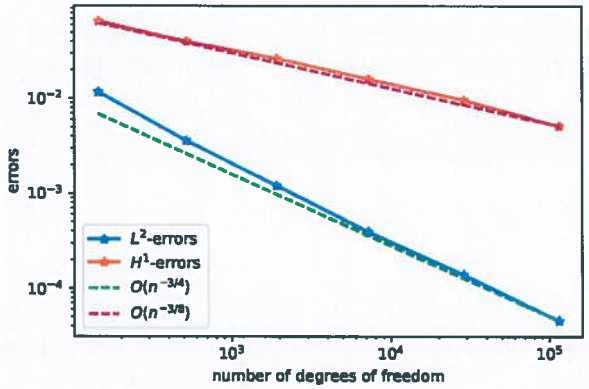


- no H^2-regularity anymore due to nonsmooth/non-convex domain (cf. Ex. 4.3)

↳ rates worse than for H^2-regular problems

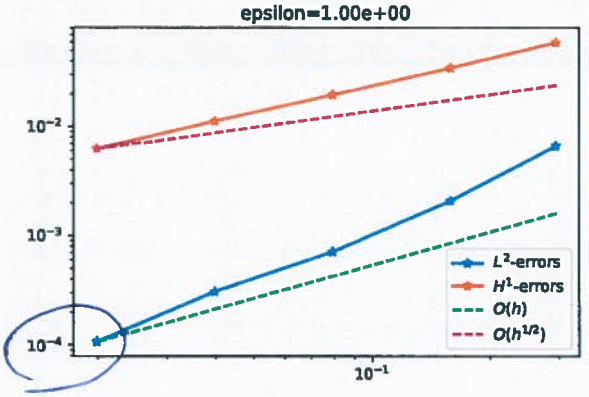
Piecewise quadratic FEM

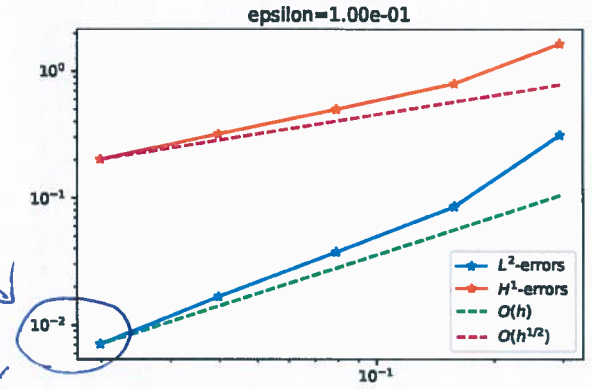


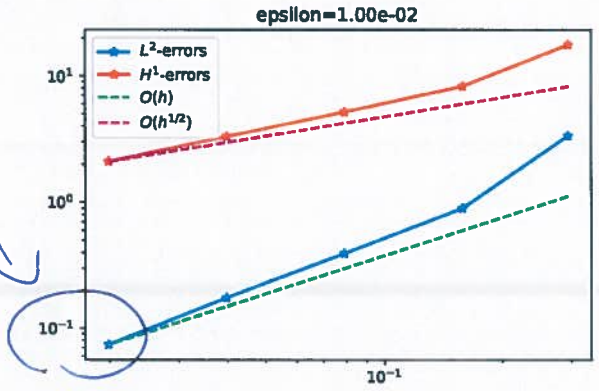


Due to missing higher regularity of the solution application of pcw quadratic FEM does not improve the H^1-convergence rate now.

Programming exercise 10.1b): Discontinuous coefficients







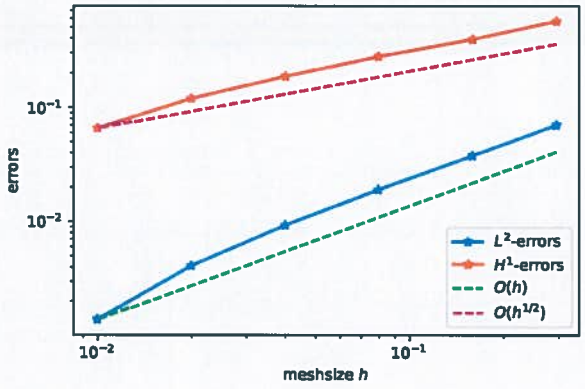
- again, no H^2-regularity ($H^{3/2}$ is the best we can expect here and the rates look like $H^{3/2}$-regularity ...)

↪ rates worse than in the lecture

- For fixed mesh size h errors grow with increasing "contrast" of the coefficient ($\varepsilon \downarrow 0$)

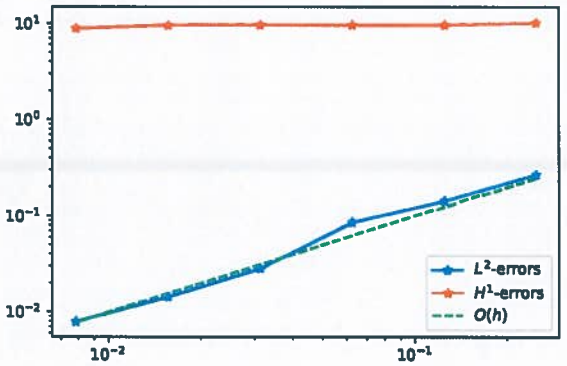
Remark:
"Faster" convergence in the "last step" is due to the inexact error evaluation based on reference solutions

Programming exercise 10.1b): Mixed boundary conditions



- no H^2-reg. in case of mixed boundary conditions (cf. Ex. 4.2)
- The problem is not only due to the discontinuity in the Neumann bc's but in the change from Neumann to Dirichlet bc's

Programming exercise 10.1c): Measure right hand side



- L^2-rate is as expected from Ex. 9.2.
- no convege w.r.t. H^1-norm, because solution is not H^1-regular!