Axioms of Adaptivity: Rate optimality of adaptive algorithms with separate marking

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Mixed finite element methods with flux errors in $H(\mathrm{div})$-norms and div-least-squares finite element methods require the separate marking strategy in obligatory adaptive mesh-refining. The refinement indicator $\sigma_\ell^2(K) = \eta_\ell^2(K) + \mu^2(K)$ of a finite element domain K in a triangulation $\mathcal{T}_\ell$ on the level ℓ consists of some residual-based error estimator η_ℓ with some reduction property under local mesh-refining and some data approximation error μ_ℓ. Separate marking (Safem) means either Dörfler marking if $\mu_\ell^2 \leq \kappa\eta_\ell^2$ or otherwise an optimal data approximation algorithm run with controlled accuracy as established in [CR11, Rab15] and reads as follows

for $\ell = 0, 1, \ldots$ **do**
  Compute $\eta_\ell(K)$, $\mu(K)$ for all $K \in \mathcal{T}_\ell$
  if $\mu_\ell^2 := \mu^2(\mathcal{T}_\ell) \leq \kappa\eta_\ell^2 \equiv \kappa\eta_\ell^2(\mathcal{T}_\ell)$ **then**
    $\mathcal{T}_{\ell+1} := \texttt{Dörfler\_marking}(\theta_A, \mathcal{T}_\ell, \eta_\ell^2)$
  else
    $\mathcal{T}_{\ell+1} := \mathcal{T}_\ell \oplus \texttt{approx}(\rho_B\mu_\ell^2, \mathcal{T}_0, \mu_\ell^2)$.

The enfolded set of axioms (A1)–(A4) and (B1)-(B2) plus (QM) simplifies and generalizes [CFPP14] for collective marking, treats separate marking in an axiomatic framework for the first timex, generalizes [CP15] for least-squares schemes, and extends [CR11] to the mixed FEM with flux error control in $H(\mathrm{div})$.

The presented set of axioms guarantees rate optimality for AFEMs based on collective and separate marking and covers existing literature of rate optimality of adaptive FEM. Separate marking is necessary for least-squares FEM and mixed FEM with convergence rates in $H(\mathrm{div}, \Omega) \times L^2(\Omega)$.

References

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