Finite difference computation of the permeability of textile reinforcements with a fast Stokes solver and new validation examples

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Abstract. For the simulation of the impregnation process of Resin Transfer Moulding, the permeability of the textile is a key input parameter. Using Darcy's law, the permeability can be derived from a numerical simulation of the fluid flow for a unit cell problem. In this paper we present the results of simulations with a Stokes solver, implemented in the permeability predicting software FlowTex. The results are compared with those of a Navier-Stokes solver and validated using theoretical results for model problems and with experimental data for real textiles.

Keywords: Permeability, Composites, Textile Reinforcements, Finite Difference, Stokes
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INTRODUCTION

Simulation tools for the injection process of LCM (Liquid Composite Moulding), e.g. PAM-RTM [1] or LIMS [2], allow Computer Aided Design (CAD) of the moulds for the manufacturing of composite materials. As input for the simulation, these tools require the assignment of the permeability values of the composite textile, at different positions of the preform model. Experimental measurement of textile permeability is possible, but is time and resource consuming [3]. Therefore, reliable and fast numerical computation of the permeability tensor of textiles is required.

A first step in the permeability simulation process, is the characterisation of the textile reinforcement model. For the creation of the textile models, we use the WiseTex software package [4], which includes NoWoTex for the creation of models of non-woven fabrics, and LamTex [5] for building the geometry model of a multi-layered reinforcement. The textile models are exported to a voxel description, which is the input for the permeability prediction software, FlowTex. In previous publications [6, 7, 8] we presented results of our permeability predicting software, based on the solution of the Navier-Stokes equations. Numerical results and experimental validation for woven fabrics and for a specially designed validation structure were given.

In this paper we present results of permeability computations with a new finite difference Stokes solver and we compare them with earlier results and with experimental data. The discretised linear Stokes equations are solved by an iterative method with preconditioning. The resulting solver is faster than the Navier-Stokes solver which uses time stepping to reach the steady state solution. Further, we have implemented the boundary conditions in the Stokes solver in a way that is well suited to deal with a geometry with very fine structures, as is the case for non-woven fabrics.

MATHEMATICAL MODEL

The permeability tensor $\underline{\underline{K}}$ is defined by Darcy's law

$$\langle \vec{u} \rangle = -Re\, \underline{\underline{K}}\, \langle \nabla P \rangle \,. \tag{1}$$

Here, Re denotes the Reynolds number, $\vec{u} = u(x,y,z)$ the fluid velocity and $P = P(x,y,z)$ the pressure. $\underline{\underline{K}}$ is the permeability tensor of the porous medium and $<>$ denotes volume averaging. For the computation of the velocity field and pressure distribution, we simulate the fluid flow in a unit cell of the textile model, as textiles have a periodic pattern. In the case that the fluid model is limited to creeping, single-phase, isothermal, unidirectional saturated flow of a Newtonian fluid, the flow in-between the yarns is described by the incompressible Navier-Stokes equations

$$\left\{ \begin{array}{l} \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla P + \frac{1}{Re}\Delta \vec{u} \\ \nabla \cdot \vec{u} = 0 \end{array} \right. . \tag{2}$$

To determine the permeability from (1), the solution of (2) must be computed. For numerical reasons, we do not compute the steady state directly, but we use a timestepping procedure. For low Reynolds number Re, the nonlinear convection term can be neglected. Further, since we are only interested in the steady state solution, the time derivative can be omitted, leading to the incompressible (steady state) Stokes equations.

$$\left\{ \begin{array}{c} -Re\, \nabla P + \Delta \vec{u} = 0 \\ \nabla \cdot \vec{u} = 0 \end{array} \right. . \tag{3}$$

Discretisation of the linear system (3) leads to a linear algebraic system, that can be solved by a direct or an iterative solver. Textiles are hierarchically structured materials and, in case of porous yarns, the mathematical model must consider inter- and intra-yarn flow. In previous work [6] we have shown how we deal with the intra-yarn flow. In this paper yarns are treated as solid material.

IMPLEMENTATION

The implementation of the permeability predicting software FlowTex is based on the freely available finite difference Navier-Stokes solver, NaSt3DGP, developed at the Institute for Numerical Simulation at the University of Bonn [9, 10]. For the solution of the Navier-Stokes equations (2), NaSt3D uses the Chorin projection method on a staggered grid. In the staggered grid approach, the pressure is discretised at the centre of the cells, while the velocities are discretised on the edges. This avoids the occurrence of non-physical oscillations in the pressure.

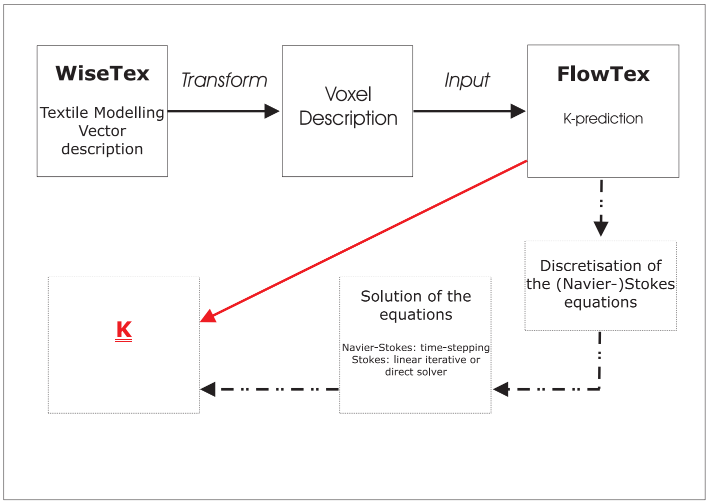

FIGURE 1. Overview of the numerical procedure

On the boundary between grid points in the fluid region and points in the solid region, no-slip boundary conditions are set. From numerical point of view, this can be implemented in two ways:

- boundary values are set explicitly in the solid cells which are bordered by fluid cells;
- the boundary conditions are included in the equation to be solved in the boundary points in the fluid region.

NaSt3D sets the boundary values explicitly. On a staggered grid, a solid point must be bordered by at least one other solid point in each direction. When the solid region forms very fine structures, as is the case for non-wovens fabrics, this constraint leads to a very fine mesh (finer than required to capture the geometry itself and to obtain a sufficiently accurate solution). More implementation details, adaptations and validation results are discussed in [6, 7, 8].

The implementation of the finite difference Stokes solver shares the interface with NaSt3D, but uses the PETSc library [11, 12, 13] to solve the resulting system of discretised equations. The Stokes solver uses a collocated grid and includes the boundary conditions into the discrete system matrix. Discretisation of (3) leads to a saddle-point problem which can be written as

$$\begin{bmatrix} \Delta & \nabla^T \\ \nabla & 0 \end{bmatrix} \begin{bmatrix} U \\ P \end{bmatrix} \approx \begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}, \tag{4}$$

with A the diffusion matrix, B the mass conservation matrix, C equals 0 or contains stabilisation coefficients and f the external force on the system. System (4) is a linear system, and the PETSc library provides several iterative and direct methods to solve the equations. The Generalised Minimal Residual Method (GMRES) with an incomplete LU (ILU) preconditioner was used for the experiments in this paper. Solving (4) with $C = 0$ results in slow convergence to an unstable solution for P. Therefore, stability

TABLE 1. Permeability values and computation time for the Natte textile

$\Delta x(mm)$	$K_{xx}(mm^2)$	**Comp. Time NaSt3D**	**Comp. Time Stokes**
Experiment	$2.27E-04 \pm 10\%$		
0.02	$3.38E-04$	16 min 00 sec	00 min 46 sec
0.03	$3.47E-04$	03 min 40 sec	00 min 26 sec
0.04	$4.38E-04$	01 min 11 sec	00 min 05 sec

coefficients are added to the matrix C. We have chosen for the unscaled stability matrix of the *lowest order approximation* stabilisation methodology found in [14, 15].

The complete numerical procedure is summerised in Figure 1.

VALIDATION

Monofilament fabric

The Monofilament fabric Natte 2115 is a realistic structure which is close to actual textile reinforcements, and for which the permeability is determined experimentally. The full description of the Monofilament Fabric Natte 2115 test-fabric can be found in [16, 3].

Figure 2 shows the computed permeability values for the Natte model for different discretisations. Table 1 presents the computed permeability and the computation time, both for the Navier-Stokes and the Stokes solver. The computed permeability is the same for both solvers, and converges to a value close to the experimental value when the mesh is refined. Note that these calculations are performed on a single layer model of the textile. If a two-layered model is used, with random or maximum nesting, the computed permeability value is lower and is closer to the experimental value [17].

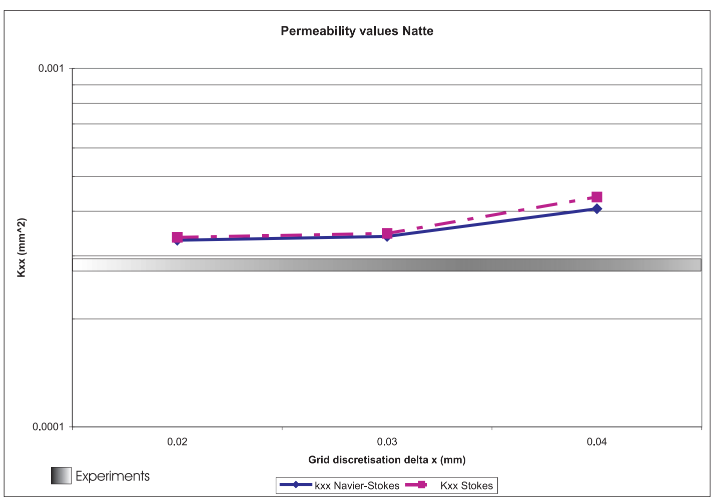

FIGURE 2. Computational results for the Natte textile

CONCLUSIONS

In this paper, we presented results of permeability calculations using the FlowTex software package. The package contains finite difference Navier-Stokes and Stokes solvers.

Using a textile model, build with the WiseTex software, flow simulations are performed to predict the permeability. Both methods were validated for a monofilament fabric Natte, for which experimental data are available. The solvers give accurate results, and with the development of the Stokes solver, a substantial reduction of computation time has been achieved. Furthermore, the Stokes solver is suited for fine structures e.g. non-woven materials. Further research on the validation for these fabrics is now possible.

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