A knowledge-based surrogate modeling approach for cup drawing with limited data

L Morand1*, D Helm1, R Iza-Teran2 and J Garcke2,3

1Fraunhofer Institute for Mechanics of Materials IWM, Wöhlerstraße 11, Freiburg 79108, Germany
2Fraunhofer Center for Machine Learning and Institute for Algorithms and Scientific Computing SCAI, Schloss Birlinghoven, Sankt Augustin 53754, Germany
3Institute for Numerical Simulations, University of Bonn, Endenicher Allee 19b, Bonn 53115, Germany

*lukas.morand@iwm.fraunhofer.de

Abstract. To predict the quality of a process outcome with given process parameters in real-time, surrogate models are often adopted. A surrogate model is a statistical model that interpolates between data points obtained either by process measurements or deterministic models of the process. However, in manufacturing processes the amount of useful data is often limited, and therefore setting up a sufficiently accurate surrogate model is challenging. The present contribution shows how to handle limited data in a surrogate modeling approach using the example of a cup drawing process. The purpose of the surrogate model is to classify the quality of the drawn cup and to predict its final geometry. These classification and regression tasks are solved via machine learning methods. The training data is sampled on a relatively wide range varying three parameters of a finite element simulation, namely sheet metal thickness, blank holder force, and friction. The geometrical features of the cup are extracted using domain knowledge. Besides this knowledge-based approach, an outlook is given for a data-driven surrogate modeling approach.

1. Introduction
A major vision driving the digital transformation in manufacturing is to transform conventional machines to autonomous systems [1]. A first step towards autonomous production systems is to couple process control with a real-time process model that infers process outcomes. Such a process model can be realized via a statistical model calibrated to process data, a so-called surrogate model. However, process data often lacks of variance because traditional manufacturing processes are designed to operate inside a predefined process window. Therefore, the information contained in the collected data is limited. To increase the information about the process, numerical simulations can be used to explore the space of process parameters outside the usual process window. However, for complex processes, numerical simulations are computationally expensive and therefore, the amount of information is also limited.

The key to improve model accuracy is to integrate domain knowledge in the modeling process. Domain knowledge can be integrated by incorporating data that is derived by a deterministic model [2, 3], and/or by deriving significant features that better represent the underlying problem as the raw data [4]. Here, the process of creating new features is either called feature learning, when performed by an
algorithm, or feature engineering, when done manually. Having meaningful features not only improves the model accuracy, but also helps the user to better interpret predictions. Besides interpretability, another important aspect is the quantification of uncertainty. For this purpose, probabilistic models can be used [5]. In manufacturing, both, the interpretability of predictions and the quantification of uncertainty, are essential to make decisions in a process and assess their risks.

In the present work it is shown how to set up a surrogate model for a cup drawing process having only a limited amount of data. Cup drawing is a testing procedure for the formability of sheet metals [6]. Various experimental and numerical studies investigate the influence of process parameters, tool dimensions, and sheet metal properties on the process outcome; see [7, 8, 9, 10].

In the following, we explain how domain knowledge can be integrated in a statistical modeling workflow via feature engineering. Also, we briefly describe the methods used for surrogate modeling and the finite element simulation of cup drawing. Then, we use the described methods to model the cup drawing process, based on data generated by the finite element simulation. Finally, concluding remarks on the knowledge-based approach are given, as well as an outlook on a data-driven surrogate modeling approach including feature learning.

2. Methods

2.1. Integrating domain knowledge via feature engineering

In a statistical modeling workflow feature engineering is part of data preprocessing. In the following, we use the cross-industry standard process for data mining CRISP-DM [11] as a reference workflow. In CRISP-DM the feature engineering task is named construct data and is located in the data preparation step, see Figure 1. The aim of the task is to derive features (i.e. pressure) that are constructed from one or more existing features (i.e. force and area) in the same data set in order to get a better description of the actual problem. In addition to the new features, a description should be given on why and how these features are derived.

Data preparation can generally not be seen as a stand-alone process. It is closely (and iteratively) connected to the previous and subsequent steps in the modeling workflow. Features are therefore highly depending on the defined modeling goal, business and data understanding, and on the machine learning model to apply and its deployment [11]. In manufacturing, feature engineering furthermore depends on knowledge about the product, its properties and technically reachable process states, the underlying physical laws, material behavior, and physically reachable states.

2.2. Surrogate modeling based on computer simulations

In this section, the employed statistical learning methods are described briefly. Related work on surrogate modeling of deep drawing simulations can be found in [12] using Gaussian processes and in [13] using neural networks. For an overview on the metamodeling of computer simulations, see [2, 14, 15]. For statistical modeling, we use the python library scikit-learn [16].

2.2.1. Random forest classification. A random forest classifier is an ensemble of randomized decision tree classifiers [17]. A decision tree classifier is a model that partitions the input space according to optimize a specific measure, i.e. information gain. This partitioning then determines the class membership for a given data point. In random forests, a number of bootstrap samples is drawn from
the data set and used to train the decision trees. Furthermore, the decision tree splits are performed using only a random subset of all features. To classify a given data point, every tree in the random forest is evaluated and the majority of votes yields the class membership [18].

2.2.2. Gaussian process regression. A Gaussian process can be seen as a distribution over functions. It is completely specified by its mean function \( m(x) \) and covariance function \( k(x,x') \) of a process \( f(x) \) [19]:

\[
m(x) = \mathbb{E}[f(x)], \\
k(x,x') = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))],
\]

where \( x \) and \( x' \) describe different locations. The Gaussian process can be written as

\[
f(x) \sim \mathcal{GP}(m(x), k(x,x'))
\]

The choice of covariance function defines the function distribution and is therefore important for the accuracy. The Gaussian process model infers probability distributions of unknown function values \( f_* \) at location \( x_* \), by deriving the joint distribution of \( f_* \) and the known function values \( f \) at location \( x \):

\[
\begin{bmatrix} f \\ f_* \end{bmatrix} \sim \mathcal{N} \begin{bmatrix} \mu, & \begin{bmatrix} K(x,x) & K(x,x_*) \\ K(x_*,x) & K(x_*,x_*) \end{bmatrix} \end{bmatrix}
\]

Equation (3) describes the noise-less case, which is sufficient for the use in this work, as only simulation data is analyzed. To include real process data it is suggested to expand equation (3) to incorporate noisy observations. Gaussian processes can be used efficiently only for a limited amount of data, as the computational complexity increases cubically with the number of data points.

2.3. Finite element simulation of cup drawing

A finite element simulation of the cup drawing process is set up with the following dimensions: The punch radius is 50 mm, the cup drawing depth is 101 mm, the radius of the sheet metal is 100 mm; cf. [7]. The velocity of the punch is kept constant during forming. The sheet metal is discretized with shell elements. To describe the material, an elasto-plastic model is used that is calibrated to experimental measurements of a DP600 dual-phase steel [20]. To define plastic flow, the Hill’48 anisotropic yield criterion is used with isotropic hardening [21]. A damage initiation criterion is applied, based on the forming limit curve for DP600 taken from [22].

Figure 2. Snap-shots of the finite element simulation showing (a) the initial state and (b) the state when the cup is drawn. Color coded is the local major (maximum in-plane principle) strain in percentage compared to the forming limit. (c) shows the location of the cylindrical coordinate system.
The simulation is performed via the finite element software Abaqus/Explicit. See Figure 2(a) and (b) for an illustration of the process. Due to symmetry, only one quarter of the cup is simulated. A cylindrical coordinate system is introduced; see Figure 2(c). The symmetry boundary conditions for displacement and rotation are: $\bar{u}_\phi = 0$, $\bar{r}_r = 0$, and $\bar{r}_z = 0$.

3. Cup drawing surrogate

3.1. Modeling aim and parameter space

In this example, the variable parameters to control the process are blank holder force and lubrication between sheet metal and tools. Apart from the tool dimensions, these parameters influence the final cup geometry and cause defects in certain combinations, cf. [9, 10]. Additionally, a strong fluctuation of sheet metal thickness during the process is considered and therefore, the parameter space is widely explored. The aim of the surrogate model is to predict the geometry of the drawn cup, given blank holder force, friction coefficient, and sheet metal thickness. In this work, the focus lies on predicting the geometry, because the final geometry of a cup is easier to measure than for example stress and strain.

Via finite element simulations, a data base is generated consisting of parameter sets and corresponding cup geometries. In case of initiating damage, the corresponding cup geometry is taken at the time increment, when major strain reaches the forming limit. The data base contains 100 training samples distributed via Latin hypercube design and 100 random test samples. Table 1 lists the parameter ranges.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>max</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>blank holder force</td>
<td>10000</td>
<td>70000</td>
<td>N</td>
</tr>
<tr>
<td>friction coefficient</td>
<td>0.01</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>sheet thickness</td>
<td>0.1</td>
<td>1.0</td>
<td>mm</td>
</tr>
</tbody>
</table>

3.2. Geometrical features

The geometry of each sample in the data base is described by a cloud of 3044 nodal coordinates. This is far too complex compared to the limited amount of training samples. Therefore, new features are needed that describe the cup geometry more compactly. A first look in the data reveals that three characteristic cup geometries are typically formed. These are defect-free cups, cups with wrinkles, and teared cups that get stuck between blank holder and die. Three characteristic geometries are exemplary shown in Figure 3.

![Figure 3. Three characteristic cup geometries. Color coded is the local major (maximum in-plane principle) strain in percentage compared to the forming limit.](image)

In the following we refer to the geometries as geometry (a), (b), and (c) as depicted in Figure 3. To distinguish geometry (c) from (a) and (b), we introduce the distance from maximal cup drawing depth to the upper $z$-coordinate in the point cloud $z_{\text{max}}$. To further distinguish (a) from (b), a measure for the smoothness of the cup wall is introduced. Therefore, the mean absolute deviation is evaluated for the outer $n$ $r$-coordinates of the sheet metal:
\[ \ddot{d} = \frac{1}{n} \sum_{i} |r_i - \bar{r}|. \] (4)

To further describe the geometry of the drawn cup, we introduce the minimal \(z\)-coordinate of the top of the cup wall \(z_{\text{min}}\). The relation between \(z_{\text{min}}\) and \(z_{\text{max}}\) can be used to describe earring in cup geometries (a) and (b). Extracting the \(\phi\)-coordinate is not necessary, as \(z_{\text{min}}\) is typically located at the boundaries of the simulated cup quarter and \(z_{\text{max}}\) is located at \(45^\circ\) from the boundaries.

### 3.3. Results and discussion

The variety of cup geometries can be visualized clearly by plotting smoothness \(\ddot{d}\) against \(z_{\text{max}}\); see Figure 4. This two-dimensional representation allows to easily distinguish between (a), (b), and (c). The cluster boundaries are set manually defining cup quality requirements.

**Figure 4.** Two-dimensional representation to distinguish the clusters (a), (b), and (c). The blue stars belong to cluster (a), the red crosses to (b), and the black triangles to (c).

As geometry (c) is far from the desired state, we can simplify the regression problem by focussing on predicting geometrical features of (a) and (b). The surrogate model is therefore realized by a two-step approach. We first filter out parameter sets leading to geometry (c) using a random forest classifier. When the random forests model expects the given parameters to produce cup geometries (a) or (b), a Gaussian process model predicts the probability distribution (mean value and standard deviation) for smoothness \(\ddot{d}\), \(z_{\text{min}}\), and \(z_{\text{max}}\). The mode of the distribution defines the most probable outcome and is therefore used as prediction. In case of Gaussian distributions, the mode is equal to the mean. Note that the Gaussian process model is fit to the logarithm of the smoothness, as \(\ddot{d}\) is close to zero but always positive and we therefore obtain a better prediction. In this case, the mode is no more equal to the mean of the distribution.

**Figure 5.** Plots of the three geometrical features: predicted vs. true value in the test set.
In the test set, the random forest model with 25 decision trees reaches an accuracy of 96% for classifying geometries (a) and (b), and 86% for geometry (c). The Gaussian process model is trained via cross-validation, using the product of a RBF, Matern, and rational quadratic covariance function. The scikit-learn implementation automatically optimizes the kernel hyperparameters. The prediction accuracy of the Gaussian process model for the test set is visualized in Figure 5.

Based on the predicted features $\bar{d}$, $z_{\text{min}}$, and $z_{\text{max}}$, an engineer is able to infer the final cup geometry. Besides the predicted features, the shape of the probability distribution is also of interest, as it can be used to quantify the uncertainty of the Gaussian process model. A high standard deviation indicates high model uncertainty, whereas a rather low standard deviation indicates that the model is certain about its prediction. To visualize this relation, we draw samples from the predicted distributions for two data points in the test set and plot them in a histogram; see Figure 6. The predicted standard deviation for the probability distribution of data point 1 is relatively small, which means that the model is quite certain. This yields a narrow probability density function, for which the mode is close to the true value (marked by the red line) as can be seen in Figure 6(a). For data point 2, in contrast, the model prediction shows a relatively high standard deviation that indicates high model uncertainty. This yields a rather shallow probability density function; see Figure 6(b). As simulation data is noiseless, a high model uncertainty indicates that the space close to the data point of interest is sampled sparsely. The accuracy of the model can then be increased by creating a new data point in that region via simulation [12].

![Figure 6. Histogram showing the predicted probability density for the geometrical features of two data points in the test set. The red lines mark the true values.](image)

4. Conclusion
Limited data and thus limited information is a serious problem for surrogate modeling of production processes. To assess the risk of the surrogate model making bad predictions, probabilistic models can be used that enable the quantification of uncertainty. Additionally, well-interpretable features help the user to better visualize and assess model predictions.

In the present work a surrogate modeling approach is shown at the example of cup drawing. Through feature engineering a low dimensional and well-interpretable representation of the cup geometry is derived. This low dimensional representation is the basis for the surrogate model and is used to efficiently visualize the data and discuss the results. The surrogate model is realized by a two-step approach. A random forest model is applied to classify the quality of the cup for a given set of process parameters. Furthermore, a Gaussian process model predicts probability distributions for the geometrical features that describe the geometry of the drawn cup. The shapes of these probability distributions thereby allow quantifying the uncertainty of the prediction. A high standard deviation indicates a higher model uncertainty.
This work is the basis for future work which compares feature engineering to feature learning methods in engineering problems. In general, feature learning is strongly applied in many fields and nowadays it is attracting interest from new communities. Nevertheless, in the field of metal forming, feature learning is still quite unpopular, which gives rise to analyze its applicability. Cup drawing is chosen to be a benchmark process, as it has been analyzed over decades and therefore typical features and their influences are well-known. In the following section, an outlook is given on feature learning approaches.

5. Outlook
Feature engineering is a convenient way to handle surrogate modeling for product development. Adequate defined features can represent global effects effectively reducing the dimensionality of the problem. Nevertheless, features must first be found, which is problem dependent, needs fundamental knowledge about the domain and is not always possible, for example in case of local strong variation of deformations due to nonlinearities. In those cases, feature engineering is not the optimal way to find global features. A promising approach is to automatically find such features via feature learning. Recent deep learning approaches are especially successful in finding features; unfortunately they require many simulations to achieve an excellent result. An alternative is the use of a variance based approach that can compactly represent a number of simulations, based on principal component analysis (PCA). The approach can provide surrogate models with good prediction capabilities in the convex hull of the input parameters of the training simulations, although the presence of nonlinearities and bifurcations drastically reduces its usefulness.

Alternative methods that exploit the geometry in order to compactly represent geometries with few features have been developed inspired from the area of 3D graphics [23]. In 3D graphics, one of the goals is to identify similar shapes, independent of their deformations. For this purpose, one of the most successful methods uses eigenfunctions of the Laplace-Beltrami operator [24]. The eigenfunctions represent features for the shapes, which can be used instead of the whole geometry. In another approach, such spectral representations are used to represent shapes in a multiscale way, ranging from coarse to fine mesh representations [25]. Also, pose independent representation of shapes have been studied using the eigenfunctions of the Laplace operator [26].

Useful synergies between 3D graphics and computational engineering have shown to provide promising results. In [23, 27], an approach has been introduced that compactly represents shapes and its deformations based on geometrical affine compact features. The approach has shown to provide adequate representations even in the presence of nonlinearities and bifurcations.

Acknowledgements
This work is part of Fraunhofer project Grey-Box-Modelle - Integration von Anwendungskenntnissen in Lernverfahren. We thank the Fraunhofer-Gesellschaft for funding and the project members for their support and the helpful discussions. We also thank Maria Baiker and Jan Pagenkopf for providing the finite element simulation of the cup drawing process.

References
[27] Iza-Teran R and Garcke J 2019 A geometrical method for low-dimensional representations of simulations *SIAM/ASA Journal on Uncertainty Quantification*