A physical temperature profiling method using gradient flows

B. Berkels*, U. Clarenz*, S. Crewell**, U. Löhnert***, M. Rumpf*, C. Simmer***

*University of Duisburg-Essen, Lotharstraße 65, 47048 Duisburg, Germany
e-mail: [bb, clarenz, rumpf]@math.uni-duisburg.de
**University of Munich, Theresienstrae 37, 80333 Munich, Germany
e-mail:screwell@uni-bonn.de
***University of Bonn, Auf dem Hügel 20, 53121 Bonn, Germany
e-mail:[csimmer, uloeh]@uni-bonn.de

Abstract

We present a new mathematical technique for retrieving the temperature profile from a ground based microwave profiler and ancillary measurements. It is based on a gradient flow approach. We are able to solve the inverse problem of radiative transfer and determine the temperature profile from ten simultaneous brightness temperature measurements in the range from 50.8 to 58.8 GHz. The approach uses no additional statistical information. A physical consistent solution for a temperature profile can be found within five seconds. First results of this technique show promising results with regard to absolute accuracy and inversion height determination, even if the brightness temperature measurements are assumed noisy to 0.5 K.

1 The inverse problem of radiative transfer

A major desire for numerical weather prediction is the availability of continuously measured profiles of temperature (T), humidity (q) and pressure (p). Such measurements are of extreme importance for operational data assimilation in the sense of short term weather forecasting. However, still today operational cost- and man-power extensive radiosondes launches (two to four times a day) state the main source for atmospheric profiles used to initialize weather forecast models.

Recent technical advances in microwave technology have brought forth microwave profilers, which can simultaneously measure brightness temperatures at a multiple number of frequencies. Operation of these profilers in zenith and also at other elevation angles has the potential of deriving continuous atmospheric profiles of temperature and humidity.

Standard methods for inferring these atmospheric profiles from brightness temperature measurements are mostly on a statistical basis, however some methods with physical constraints have also been developed. A standard approach [4] uses simultaneously measured microwave and infrared brightness temperatures and radiosonde profiles of T and q to create a representative data set and invert the radiative transfer equation via multiple regression and neural networks. In [8] retrieval methods are proposed based on a Bayesian approach. Here, the *optimal estimation equations* in the formulation of Newtonian Iteration can combine measurements and statistical information in an optimal way, such that certain physical constraints are fulfilled. Such an approach to infer T, q, and cloud liquid water profiles has been successfully applied to a multiple sensor combination in [6].

We present a novel and fast numerical technique to derive temperature profiles from microwave frequencies located at the wing of the 60 GHz oxygen absorption complex. Here we describe a new mathematical approach for a T retrieval, which in contrast to many other retrieval methods, does not rely on statistical data sets, thus can be characterized as "purely physical". The advantage of such a method is that it may be applied virtually to any arbitrary site. The mathematical key tool for the solution of the problem consists in so called regularized gradient flows. For a theoretical background and applications to inverse problems we refer to [7, 9, 2, 1]

2 The gradient flow method

In our measurement configuration the multi-channel microwave radiometer is located at the ground, the atmosphere is horizontally stratified and no clouds are present. We assume that the microwave radiation measured at the ground depends only on T, q, p, elevation angle θ, and frequency ν. To compute the temperature profile $T(z)$ we take into account n brightness temperature measurements (TBM_1, ... , TBM_n) at frequencies ν_1 to ν_n. In our special case these frequencies are chosen to be 50.8, 51.8, 52.8, 53.8, 54.8, 55.8, 56.8, 57.8, and 58.8 GHz, corresponding to one band of the 22 -channel microwave profiler MICCY from the University of Bonn, Germany.

It is our goal to retrieve a realistic temperature profile $T(z)$ such that the TBM will be consistent with the forward model brightness temperatures. In a first approximation we assume that all atmospheric parameters except the temperature are given. We consider the quadratic cost functional $E[T]$ whereby the task is to find a temperature profile $T(z)$ such that $E[T]$ is minimal. It is obvious that this is an ill-posed problem and that many irregular temperature profiles are solutions. Let us now describe in more detail the regularization and the numerical algorithm behind our method. In our special case the temperature profile appears as one parameter steering the brightness temperature TB. The following representation is a well known formulation of the radiative transfer equation [5] for a horizontal homogeneous and non-scattering atmosphere in case the radiometer is located at the ground:

$$\begin{aligned} &TB(\nu,\theta) = \\ &TB_0 \exp\left(-\sec(\theta)\int_0^\infty \sigma_a(\nu,T,q,p)\,dz\right) \\ &+\sec(\theta)\int_0^\infty \sigma_a(\nu,T,q,p)\,T * \\ &\exp\left(-\sec(\theta)\int_z^\infty \sigma_a(\nu,T,q,p)\,dz'\right)\,dz\,. \end{aligned} \tag{1}$$

In the above equation z indicates the altitude, TB_0 the cosmic background of 2.73 K and σ_a the volume absorption coefficient.

Let us introduce the vector $Q(z) = (q(z), p(z))$. In the following discussion we assume that this vector $Q(z)$ is known for all z. In this sense we want to present a method which is able to compute the temperature profile $T(z)$. Furthermore we fix $\theta = 90^o$ in our considerations. We notice that the above integral formula for $TB(\nu)$ is equivalent to the ordinary differential equation

$$\frac{d}{dz}TB(\nu,z) = G(TB(z),\nu,T(z),Q(z)) \tag{2}$$
$$TB(\nu,\infty) = TB_0\,, \tag{3}$$

where the right hand side is given by

$$G(TB,\nu,T,Q) = \sigma_a(\nu,T,Q)\,TB - \sigma_a(\nu,T,Q)\,T\ .$$

Assume we have data $TBM_1,\ldots,TBM_n$ for n frequencies $\nu_1,\ldots,\nu_n$. We want to determine a regular temperature profile $T(z)$, such that for all frequencies ν_i we have $TBM_i = TB(\nu_i)[T]$, where we set $TB(\nu_i)[T] := TB(\nu_i, 0)$. The quadratic cost functional for this problem is

$$E[T] = \frac{1}{2}\sum_{i=1}^{n}(TBM_i - TB(\nu_i)[T])^2\,. \tag{4}$$

In the following we describe in more detail the computation of $E'[T]$ which is given by

$$\langle E'[T],\delta T\rangle = \lim_{\epsilon\to 0}\frac{E[T+\epsilon\,\delta T]-E[T]}{\epsilon}\,.$$

By the definition of E we obtain:

$$\begin{aligned} \langle E'[T],\delta T\rangle = -&\sum_{i=1}^{n}(TBM_i - TB(\nu_i)[T]) * \\ &\lim_{\epsilon\to 0}(TB(\nu_i)[T+\epsilon\,\delta T] - TB(\nu_i)[T])/\epsilon\,, \end{aligned}$$

and our problem is reduced to the evaluation of

$$\lim_{\epsilon\to 0}\frac{TB(\nu_i)[T+\epsilon\,\delta T]-TB(\nu_i)[T]}{\epsilon}\,.$$

This can be achieved using the linearization of the ODE (2). This linearization leads to the ODE

$$\begin{aligned} \frac{d}{dz}\Delta(\nu_i,z) = &\,\partial_T G(TB(z),\nu,T(z),Q(z))\,\delta T(z) + \\ &\partial_{TB}G(TB(z),\nu,T(z),Q(z))\,\Delta(z) \end{aligned} \tag{5}$$
$$\Delta(\nu_i,\infty) = 0\,. \tag{6}$$

Computing a solution of this differential equation enables us to compute the derivative E' via

$$\lim_{\epsilon\to 0}\frac{TB(\nu_i)[T+\epsilon\,\delta T]-TB(\nu_i)[T]}{\epsilon} = \Delta(\nu_i,0)\,.$$

Let us now explain how a gradient flow method can be applied to solve the inverse problem to determine $T(z)$. The idea is to introduce a regularizing metric g measuring the derivative of E in a regular space $\mathcal{V}$. If we consider the duality in $\mathcal{V}'$ we have a representation $A : \mathcal{V} \to \mathcal{V}'$ of g :

$$g(u,v) = \langle Au, v\rangle\,.$$

Obviously, this mapping is bijective on account of the metric properties. If we measure the derivative w.r.t. g then the formal gradient flow

$$\partial_s T(s) = -\mathrm{grad}_g E[T(s)]$$

with respect to the metric $g(\cdot,\cdot)$ reads as $g(\partial_s T,\varphi) = -\langle E'[T],\varphi\rangle$, for all $\varphi \in \mathcal{V}$. This can be re-formulated using the mapping A by $A\partial_s T = -E'[T]$ or equivalently:

$$\partial_s T = -A^{-1}E'[T]$$

and the mapping A^{-1} transfers the derivative of E into the space $\mathcal{V}$. For more details concerning the relation between this interpretation of a gradient flow and regularizing energies we refer to [3].

3 Practical aspects

In our applications the choice of the above mapping A is simply such that A^{-1} is some standard filter. Note that it is possible to incorporate at this point better suited metrics, if we have additional a priori assumptions on the temperature profiles.

The solution of the linearized ODE (5),(6) is obtained by a standard Runge-Kutta method. To make the approach efficient, we have additionally used an Armijo-like timestep-control.

The obtained T profiles are piecewise linear with typically 5 to 7 points leading to 10 to 14 degrees of freedom. Figure 1 shows an example of our technique applied to a December temperature profile from Netherlands. The ten brightness temperatures were calculated with a standard radiative transfer model and subsequently forseen with a Gaussian noise of 0.1, 0.3 and 0.5 K. The temperature profile shows a sharp inversion from 1.6 to 1.9 km and an approximately isothermic condition from 2 to 3 km height. In our case we assume the temperature is given in the lowest 200 m due to continuous temperature measurements (200 m mast) at the location of the MICCY radiometer (Cabauw, NL). Humidity, pressure, and temperature above 8 km are also assumed to be known in this first study. Notice that inversion height and the range of the isothermic condition are accurately recovered even with noisy errors up to 0.3 K. In this case the absolute accuracies below 3 km are on the order of 1 K or less. Let us point out that one retrieval is possible within approximately 5 seconds (with a non-optimized code on a standard PC).

Currently we are applying our method to a wide range of atmospheric conditions. One of the next steps consists is including elevation information, which should also enable us to integrate this method into a suitable multiscale with finer resolution at lower altitude. Furthermore, we will incorporate a backward computation for humidity.

References

[1] S. Angenent, S. Haker, and A. Tannenbaum. Minimizing flows for the Monge-Kantorovich problem. *SIAM J. Math. Anal.*, 35(1):61–97 (electronic), 2003.

[2] U. Clarenz, M. Droske, and M. Rumpf. Towards fast non–rigid registration. In *Inverse Problems, Image Analysis and Medical Imaging, AMS Special Session Interaction of Inverse Problems and Image Analysis*, volume 313, pages 67–84. AMS, 2002.

[3] U. Clarenz, S. Henn, M. Rumpf, and K. Witsch. Relations between optimization and gradient flow methods with applications to image registration. In W. Hackbusch and M. Griebel, editors, *Multigrid and related methods for optimization problems*, 2002.

[4] J. Güldner and D. Spänkuch. Remote sensing of the thermodynamic state of the atmospheric boundary layer by ground-based microwave radiometry. *J. Atmos. Oceanic Technol.*, 18:925–933, 2001.

[5] M. A. Janssen. *Atmospheric Remote Sensing by Microwave Radiometry*. John Wiley and Sons, Inc., New York, 1993.

[6] U. Löhnert, S. Crewell, and C. Simmer. An integrated approach towards receiving physically consistent profiles of temperature, humidity, and cloud liquid water. *J. Appl. Meteor.*, to appear.

[7] F. Otto. The geometry of dissipative evolution equations: the porous medium equation. *Comm. Partial Differential Equations*, 26(1-2):101–174, 2001.

[8] C. D. Rodgers. *Inverse Methods for Atmospheric Sounding - Theory and Practice*. World Scientific, Singapore, 2000.

[9] O. Scherzer and J. Weickert. Relations between regularization and diffusion filtering, 1998.

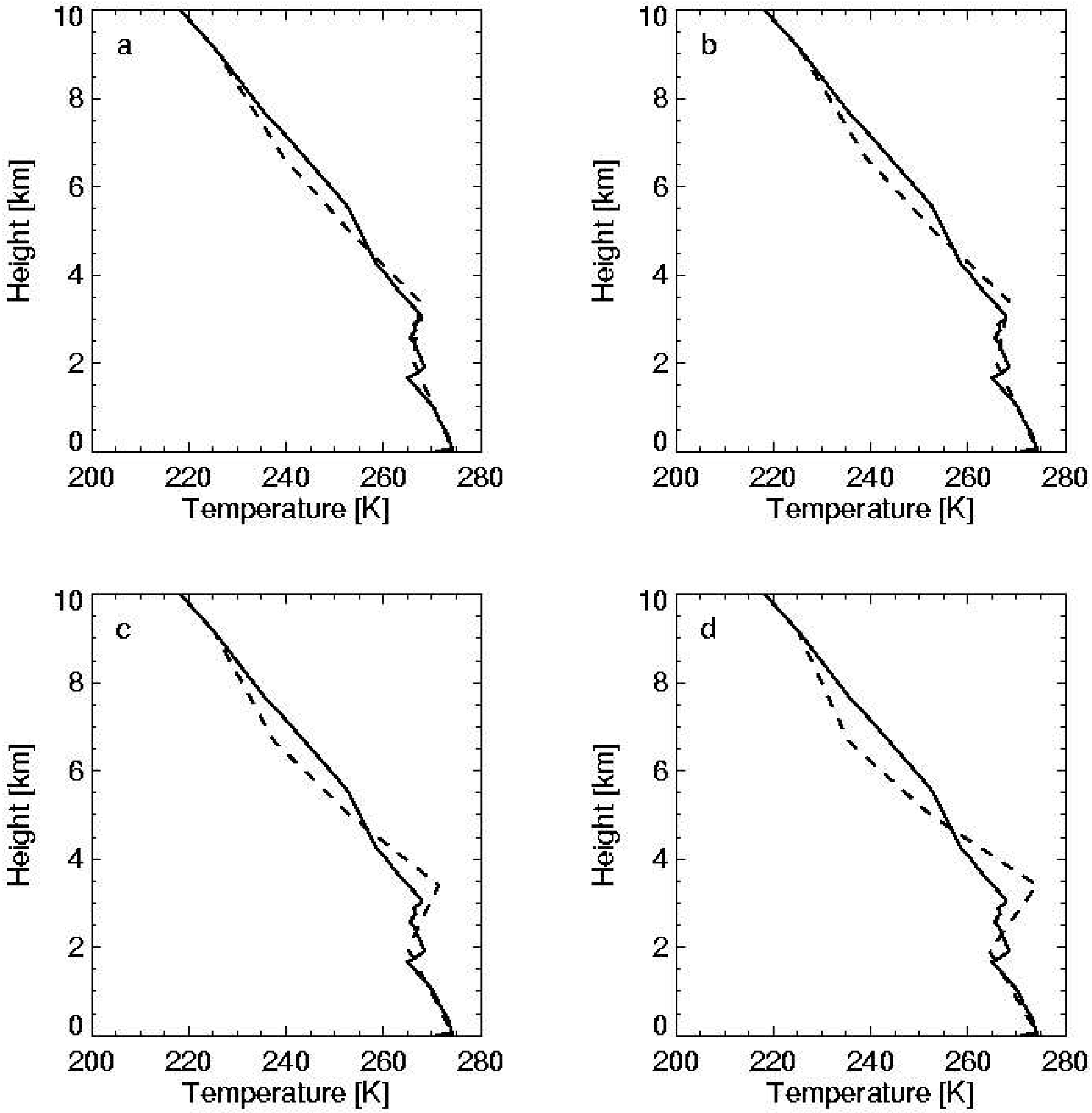

Figure 1: Retrieved temperature profile (dashed) with the method of gradient flows (a). The continuous line shows the original radiosonde profile (same in all four panels). Panels (b)-(d) show the retrieval results if the brightness temperatures are forseen with Gaussian noise of 0.1, 0.3, and 0.5 K, respectively.