Corrections

The following is a list of corrections of typographical, notational, mathematical, and other errors and mistakes in the text that were discovered after the manuscript was submitted. Readers are encouraged to contact the authors if they find other inconsistencies and errors, or if they have suggestions for improvement. The solutions to the problems in the book are available from the authors on request.

- p.36, line -1: It should be p_i, not $\mathbf{p}_i$, in the formula.
- p.64, line -7: The problem heading should read "Stable subspace splittings ...".
- p.106, Fig. 3.5 a): The notation attached to the graph should be ϕ, not ψ.
- p.111, line -7 ff: From the formula (3.34) on page 111 to the formulation of Theorem 3.5 on page 113, the previously introduced notation $N_{P,j}$ for finite element nodal basis functions of level j changes to $N_{j,P}$, which is a notational inconsistency.
- p.160, line 3: It should be $\tilde{a}_\rho$, not $\tilde{a}$, in the denominator of the formula.
- p.167, lines 9 and 10: Drop the subscript 2 in the norms.
- p.179, line -12: Replace "With this choice of I_m, we have" with "With this choice of I_m, for the iteration (4.1) with constant step-size parameter $\omega_m = \lambda_{\max}^{-1}$, we have".
- p.184, line -1: It should be $e_w^{(m)}$, not $e_w^{(m+1)}$.
- p.185, line 8: It should be $a(P_\rho^{-1}(\delta_m e_w^{(m)} + (1-\delta_m)e_v^{(m)}), d_v^{(m)})$ or, alternatively, $a(\delta_m e_w^{(m)} + (1-\delta_m)e_v^{(m)}, P_\rho^{-1} d_v^{(m)})$.
- p.185, line -7: Obsolete (repeated from the previous line).
- p.192, line 1: Replace r_m with r'_m.
- p.197, Problem 4.3: Delete Part b) here. This has already been established in Part b) of Problem 2.1.
- p.197, line 14: It should be $W_i^\perp = \mathrm{ran}(R_i)$, not W_i. This is because $\cos(\angle(W_i, W_j)) = \cos(\angle(W_i^\perp, W_j^\perp))$ only if $W_i \cap W_j = \{0\}$.
- p.197, line -16, formula (4.108), and p.198, line 1: It should be the operator norm $\|M_{\rho,\omega}\|_V$ instead of $\|M_{\rho,\omega}\|$.

- p.198, line 4: Replace for clarity with

$$u_1 = Q_{W_i^\perp} u, \quad u_2 = Q_{U_{i+1}} Q_{W_i} u, \quad u_3 = Q_{U_{i+1}^\perp} Q_{W_i} u.$$

- p.198, line 7: Replace $\|u\|^2$ with $\|u_2 + u_3\|^2$ or, better, by $(\|u_2\|^2 + \|u_3\|^2)$.
- p.198, lines -11 and -10: Replace the sentence "Find a complementing ..." with "For the upper bound, note that $\tilde{M}_{\rho,\omega} \in \mathcal{B}_{pos}(V)$."
- p.198, line -9: It should be $I - \omega C_j A$ instead of $I - C_j A$ in the product of operators.
- p.199, lines 15 and 16: Replace the formula containing $r(M_{\eta,\xi})$ in line 15 with

$$\sqrt{\eta} < r(M_{\eta,\xi}) = \begin{cases} p_{\max} + \sqrt{p_{\max}^2 - \eta}, \\ \max(p_{\min} + \sqrt{p_{\min}^2 - \eta}, p_{\max} + \sqrt{p_{\max}^2 - \eta}), \\ p_{\min} + \sqrt{p_{\min}^2 - \eta}, \end{cases}$$

  for the three cases

$$\begin{array}{l} 0 < \xi \leq (1 - \sqrt{\eta})^2/\lambda_{\min}, \\ (1 - \sqrt{\eta})^2/\lambda_{\min} < \xi < (1 + \sqrt{\eta})^2/\lambda_{\max}, \\ \xi \geq (1 + \sqrt{\eta})^2/\lambda_{\max}, \end{array}$$

  respectively, and define $p_{\min} = -(1 + \eta - \zeta\lambda)/2$ in line 16.
- p.200, lines -1 and -4: Replace β_m with δ_m three times.
- p.210, line 14: Change the norm in the denominator to $\|A^+ D^{1/2} I_{\mathbf{p}}^{-1/2} y\|_2$.
- p.211, line 8: Remove the obsolete I_ρ from the formula.
- p.213, line -1: It should be $i = 1, \ldots, n'$.
- p.215, line 1: In the matrix that appears in the formula (5.19), replace γ^{-1} with $\gamma^{-1/2}$ (two times) and γ^{-2} with γ^{-1}.
- p.228 ff: Starting with the formula in line -10, the notation for the norm $\|\cdot\|_{\tilde{V},1-s}$ is shortened to $\|\cdot\|_{1-s}$ in many places. This affects p.228 (lines -10 and -3), p.229 (lines 9,-3,-2), p.230 (lines 2,8,10,11,12), p.231 (lines 8,9,15) and p.249 (lines 8,13,18). Replace with $\|\cdot\|_{\tilde{V},1-s}$ for consistency.
- p.237, line 19: The lower bound in (5.57) should be corrected to

$$\frac{\mathbb{E}(\|e_x^{(m)}\|^2)}{\|e_x^{(0)}\|^2} \geq 1 - \frac{Cm}{n(n+1)}, \qquad m = 1, \ldots, N.$$

  In Problem 5.8 this is established with $C = 2\sqrt{e}$.
- p.246, lines -3 to -1: Ignore. We have no direct counterexample for this bound, but we cannot prove it either. Our initial sketch of a proof was flawed.
- p.247, line 6: After "... Schwarz iteration" insert the reference "(5.3)".
- p.247, line 7: Add the sentence "Furthermore, assume $i_0 = 1$."
- p.247, lines 9 and 14: The formulae in these lines should be corrected to

$$\|e^{(m+1)}\|^2 = c^{2m} \|e^{(1)}\|^2 = \frac{c^{2m}}{1 - c^2}(r_1^{(0)} - c r_2^{(0)})^2 \leq c^{2m} \|e^{(0)}\|^2$$

and

$$\mathbb{E}(\|e^{(m+1)}\|^2) = \left(\frac{1+c^2}{2}\right)^m \mathbb{E}(\|e^{(1)}\|^2) \le \frac{1+c}{2}\left(\frac{1+c^2}{2}\right)^m \|e^{(0)}\|^2,$$

respectively.

- p.249, line 2: The expression on the right-hand side of the inequality should include the term $|||\tilde{e}_u^{(0)}|||_{\tilde{V},1-s}$, which for $J \in \mathcal{F}_L^{1,1}$ is not necessarily bounded from above by a multiple of $\Delta\tilde{J}_u^{(0)}$, as erroneously stated at the end of the hints to the problem. But, for example,

$$\Delta\tilde{J}_u^{(m)} \le C\frac{\Delta\tilde{J}_u^{(0)} + |||\tilde{e}_u^{(0)}|||_{\tilde{V},1-s}}{m^2}, \qquad m = 1, 2, \ldots,$$

is a correct (though not optimal in terms of constants) replacement.

- p.249, lines -12 to -10: Replace with

$$(\alpha'_{m+1})^{-1} = 1 + \gamma_{m+1} = 1/2 + \sqrt{(\alpha'_m)^{-2} + 1/4} \ge 1/2 + (\alpha'_m)^{-2}.$$

Iterating this gives the result with a constant C depending on the quotient $n(\tilde{S}'_s)^2/\tilde{S}_s$ and the choice of $(\alpha'_0)^{-1} = 1 + \gamma_0$.

- p.249, line -5: Replace the subscript ℓ^2 with 2.
- p.250, line 2: Should be corrected to $N - \frac{j+1}{2}$.
- p.250, line 12: To match the notation in the text, it should read $e_x^{(m)}, e_y^{(m)}, e_z^{(m)} \in X_j$ instead of $e^{(m)} \in X_j$.
- p.250, line -11: In the formula, correct to

$$\ldots = N - \frac{2sr}{s+r} \ge N - \frac{s+r}{2}, \ldots$$

- p.251, line 8: Replace $2 \le \ldots$ with $2 < \ldots$
- p.251, lines 10 to 15: The hint for b) is correct in principle. But the proof is much simpler if one aims at proving

$$\mathbb{E}'(\|e^{(m+1)}\|^2) \ge q\|e^{(m)}\|^2, \quad m \ge 0,$$

with some constant $q > 1$ depending on ω and α.

- p.259, line 9: An index shift is required: Replace $\bar{u}^{(m+1)}$ and $\bar{u}^{(m)}$ with $\bar{u}^{(m)}$ and $\bar{u}^{(m-1)}$, respectively. This is consistent with the following considerations.
- p.260, lines -9 and -8: The subscripts of the norms should read V_P^t and V_P^s, respectively.
- p.269, line -1 and p.270, line 10: It should read $a(e^{(m)}, u)$, not $a(e, u)$.
- p.288, line -6: It should be $\bar{\varepsilon}_{m+1} = \alpha_m(1 - \rho_i\mu_m)\bar{\varepsilon}_m + \bar{\alpha}_m c_i$.
- p.289, line 3: The last term should be $\bar{\alpha}_{m-1}c_i$, not $\bar{\alpha}_{m-2}c_i$.
- p.289, line 6: The specification $S_0 = 1$ is missing.
- p.303, line -2: Add the natural assumption $t < s$ (or $s - t > 0$).

- p.303, line -1: The exponents in the two-sided estimate should be $-2(s-t)$ or $2(t-s)$, not $2(s-t)$. Strictly speaking, the hints for the problem on p.304 are more appropriate for the related quantity

$$\bar{\varepsilon}_m := \sup_{\|P\|\le 1} \inf_{p_m:\, p_m(0)=1} \sup_{e^{(0)}\in V_P^s} \frac{\|p_m(P)e^{(0)}\|^2_{V_P^t}}{\|e^{(0)}\|^2_{V_P^s}},$$

  which describes the best possible asymptotic convergence rate for *linear* Krylov subspace methods, while ε_m ($\le \bar{\varepsilon}_m$) is related to optimal best approximation from $\mathcal{K}_m$ and also covers nonlinear Krylov subspace methods. Currently we only have a proof of the lower bound for $\bar{\varepsilon}_m$ but not for ε_m.
- p.305. line 3: It should be $u^{(m+1)}$, not u^{m+1}.
- p.305, line 6: The denominator in the definition of γ_m should be $\sqrt{\beta_m + 1/4}+1/2$.
- p.305, line 7: The recursion should be $\sqrt{\beta_{m+1}} = \sqrt{\beta_m + 1/4} + 1/2$.
- p.305, line 10: It should be $\beta_m \ge (m+1)^2/4$, not $\beta_m \ge (m+1)^2$.
- p.306, line 12: Correct the text to read "uniformly bounded away from zero for infinitely many m, depending on the spectrum of $P_{\mathbf{p}}$."
- p.308, lines -3 and -1 and p.309, line 2: σ should be replaced with σ_H.
- p.310, line 4: It should be $m = 1, 2, \ldots$, the quantities $S_0 = S_0' = 1$ and $S_0'' = 0$ do not need to be estimated.
- p.310, line -2: In this inequality a term +1 is missing (this is because the number of integers in an interval $[a, b)$ belongs to the interval $(b-a-1, b-a+1)$ and in the worst case can be arbitrarily close to the endpoints of this interval). This +1 can be compensated by an additional constant in the following estimates. Note that the estimation of $|I_n|$ is simplified if one first observes that $|I_0| \ge |I_n|$ for all $n \ge 1$.
- p.312, line 15: In the formula it should read v_i, not $\tilde{v}_i$.
- p.312, line -4: For consistency it should be $G := (0, 1)$.
- p.313, line 5: It should be $b(\cdot,\cdot)$, not $B(\cdot,\cdot)$.
- p.313, line -4: It should read "The constants in a) and d)", not "The constants in a) and b)".
- p.313, line -2: In the meantime, we have realized that there are several (finite element and wavelet) multiscale splittings that provide answers for Part e) of the problem.
- p.334, lines -4 and -5, p.335, line 7: It should read $\hat{V}_{j,0}$, not $\hat{V}_j$ (analogously on p.336, lines 11 and -2 and on p.337, line -4).
- p.337, line -11: It should be $\hat{u}_J(M_e)$ in the formula, not $\hat{u}(M_e)$.
- p.355, line -6: The := sign can be replaced with = since $\bar{w}_{\tilde{\Gamma}}$ was already defined in line -11.
- p.373, line -4: For notational consistency, replace the subscript $L^2(G)$ with L^2.
- p.387, line -13: It should read "subspace $\hat{V}_J \subset \ldots$, and not "subspaces $\hat{V}_J \subset \ldots$, since $\hat{V}_J$ is one subspace.
- p.388, line 20: It better should read $V_{j+1,0}$ and $\hat{V}_{j,0}$, instead of $V_{k+1,0}$ and $\hat{V}_{k,0}$.
- p.389, line 14: It should be $V_{G_{ik}}$, not $V_{G_i k}$.

- p.389, Problem 7.8: We have solved this problem in a sightly different way, namely starting from the ladder

$$\bar{V}_{0,0} \subset \ldots \subset \bar{V}_{j_0,0} \subset V_{j_0+1,0} \subset \ldots \subset V_{J,0}, \qquad 0 \le j_0 < J.$$

  Then the formal definitions of V_{G_i}, $\bar{V}_{\Gamma_{ik}}$ and the vertex spaces $\bar{V}_P$ as spans of scaled nodal basis functions $\tilde{N}_{P',j}$, $j_0 < j \le J$, for nodal points P' interior to G_i, interior to Γ_{ik} and with $P' = P$, respectively, remain the same and the proofs become clearer. The differences with the original formulation of Problem 7.8 are minimal.
- p.390, lines -14 and -12: It must read

$$\frac{A}{2\max(1, A_0)} \le \bar{A} \le \bar{B} \le B$$

  and $\kappa_{\mathbf{V}} \approx (\max(1, A_0))^{-1}$, respectively.
- p.392, Fig. 7.8: For better compatibility with Figure 3.5 b), the function values shown for ψ_3 should be multiplied by 5/6, i.e., they should be -1, $11/12$, $-1/2$, $1/12$ (similar changes for ψ_4).
- p.392, line -1: Replace the word ”exponential” with ”exponent”.