NON-RIGID MORPHOLOGICAL IMAGE REGISTRATION & ITS PRACTICAL ISSUES

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ABSTRACT

We present a novel variational method to non rigid registration of multi-modal data. A suitable deformation will be determined via the minimization of a morphological, i.e., contrast invariant, matching functional along with an appropriate regularization energy. Here we want to give special focus on the practical issues involving scale-space methods, regularization of the corresponding gradient flow and hyperelastic regularization.

1. INTRODUCTION

The process of registration aims to find a deformation ϕ defined on a given domain Ω onto itself, such that an image $T : \Omega \to \mathbb{R}$ correlates well under the deformation $(T \circ \phi)$ with another given image $R : \Omega \to \mathbb{R}$. It is a very difficult and challenging problem, since the construction of similarity measures for the registration of multi modal images is very delicate. Naturally, such similarity measures can no longer depend on entities which depend only on the grey values, such as it is commonly done in the case of unimodal registration. Different imaging devices focus on different physical, chemical, functional or histological characteristics of the underlying object, which also implies that the structural content of the images may differ significantly. Additionally, due to perturbations in the imaging process, external forces or temporal changes one can often not assume the deformation to be affine and in case of intra-individual registration, the variability of the anatomy can not be described by a rigid transformation, since many structures like, e. g., the brain cortex may evolve very differently in the growing process.

Currently, multi-modal registration of images has been tackled mainly and most successfully by information theoretic approaches. In this context, the concept of maximizing the *mutual information* of a pair of images is widely known. We want to follow a different approach and assume, that at least partially, the local image structure or "morphology" is very similar between the images. So the approach presented here therefore concentrates on aligning objects correctly from one image onto the other. In [5] the mathematical foundations of the approach have been developed and existence for a class of registration functionals has been proved. Furthermore more references can be found therein. In this paper, we first want to give a review of the original method and finally to address practical issues of the method, especially in Section 5 describing the actual minimization algorithm.

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2. REGISTRATION VIA GAUSS MAPS

At first, let us define the morphology $M[I]$ of an image I as the set of level sets of I:

$$M[I] := \{\mathcal{M}_c^I \,|\, c \in \mathbb{R}\}, \tag{1}$$

where $\mathcal{M}_c^I := \{x \in \Omega \,|\, I(x) = c\}$ is a single level set for the grey value c. I.e. $M[\gamma \circ I] = M[I]$ for any reparametrization $\gamma : \mathbb{R} \to \mathbb{R}$ of the grey values. Up to the orientation the morphology $M[I]$ can be identified with the normal map (Gauss map)

$$N_I : \Omega \to \mathbb{R}^d\,;\; x \mapsto \frac{\nabla I}{\|\nabla I\|}\,. \tag{2}$$

Morphological methods in image processing are characterized by an invariance with respect to the morphology [7]. Now, aiming for a morphological registration method, we will ask for a deformation $\phi : \Omega \to \Omega$ such that

$$M[T \circ \phi] = M[R]\,.$$

Thus, we set up a matching functional which locally measures the twist of the tangent spaces of the template image at the deformed position and the deformed reference image or the defect of the corresponding normal fields.

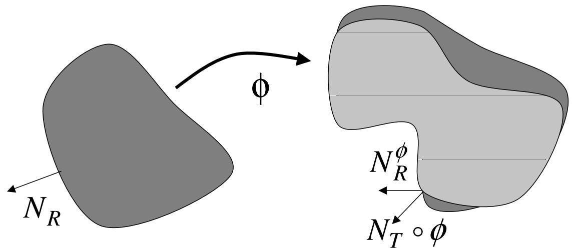

Fig. 1. Morphological defect. The isoline of R (dark gray) and normal N_R are transformed under ϕ and compared to isoline of T (light gray) and resp. $N_T \circ \phi$.

We aim to minimize a suitable matching energy, which measures the morphological defect of the reference image R and the deformed template image T, i. e., we ask for a deformation ϕ such that $N_T \circ \phi \,||\, N_R^\phi$, where N_R^ϕ is the transformed normal of the reference image R on $\mathcal{T}_{\phi(x)}\phi(\mathcal{M}^R_{R(x)})$ at position $\phi(x)$. From the transformation rule for the exterior vector product $D\phi\, u \wedge D\phi\, v = \mathrm{Cof}\, D\phi(u \wedge v)$ for all $v, w \in \mathcal{T}_x\mathcal{M}^R_{R(x)}$ one derives

$$N_R^\phi = \frac{\mathrm{Cof}\, D\phi\, N_R}{\|\mathrm{Cof}\, D\phi\, N_R\|}$$

where $\mathrm{Cof}\, A = \det A \cdot A^{-T}$ for invertible $A \in \mathbb{R}^{d,d}$ is the cofactor matrix of A, a matrix of consisting of all $(n-1)$-minors of A.

One might be tempted to define $\int_\Omega \|N_T \circ \phi - N_R^\phi\|^2 d\mu$. But, for theoretical reasons [5], we avoid the normalization appearing in N_R^ϕ and choose the following matching energy

$$E_m[\phi] := \int_\Omega g_0(\nabla T \circ \phi, \nabla R, \mathrm{Cof}\, D\phi)\, \mathrm{d}\mu. \qquad (3)$$

where g_0 is a 0-homogenous extension of a function $g : S^{d-1} \times S^{d-1} \times \mathbb{R}^{d,d} \to \mathbb{R}^+$, i. e., $g_0(v, w, A) := 0$ if $v = 0$ or $w = 0$ and $g_0(v, w, A) := g(\frac{v}{\|v\|}, \frac{w}{\|w\|}, A)$ otherwise. If we want to achieve invariance of the energy under non-monotone grey-value transformation, the symmetry condition $g(v, w, A) = g(-v, w, A) = g(v, -w, A)$ has to be fulfilled.

3. AN EXISTENCE RESULT

In this section we cite the existence result given in [5]. For the existence of a minimizer to hold, we need to control the singularities of the images and introduce the following image space

$$\mathcal{I}(\Omega) := \Big\{ I : \Omega \to \mathbb{R} \,\Big|\, I \in C^1(\bar{\Omega}), \exists \mathcal{D}_I \subset \Omega \text{ s. t. } \nabla I \neq 0 \text{ on } \Omega \setminus \mathcal{D}_I, \mu(B_\epsilon(\mathcal{D}_I)) \stackrel{\epsilon \to 0}{\longrightarrow} 0 \Big\},$$

where μ denotes the Lebesgue-measure. We then obtain the following mathematical result [5].

Theorem 1 (Existence of minimizing deformations) *Suppose $d = 3$, $T, R \in \mathcal{I}(\Omega)$, and consider the total energy for deformations ϕ in the set of admissible deformations*

$$\begin{aligned}\mathcal{A} := \{&\phi : \Omega \to \Omega \,\big|\, \phi \in H^{1,p}(\Omega), \mathrm{Cof}\, D\phi \in L^q(\Omega),\\ &\det D\phi \in L^r(\Omega), \det D\phi > 0 \textit{ a.e. in } \Omega,\\ &\phi = 1\!\!1 \textit{ on } \partial\Omega\}\end{aligned}$$

where $p, q > 3$ and $r > 1$. Suppose $W : \mathbb{R}^{3,3} \times \mathbb{R}^{3,3} \times \mathbb{R}^+ \to \mathbb{R}$ is convex and there exist constants $\beta, s \in \mathbb{R}$, $\beta > 0$, and $s > \frac{2q}{q-3}$ such that

$$W(A, C, D) \geq \beta\left(\|A\|_2^p + \|C\|_2^q + D^r + D^{-s}\right) \quad \forall A, C \in \mathbb{R}^{3,3},\ D \in \mathbb{R}^+$$

Furthermore, assume that $g_0(v, w, A) = g(\frac{v}{\|v\|}, \frac{w}{\|w\|}, A)$, for some function $g : S^2 \times S^2 \times \mathbb{R}^{3,3} \to \mathbb{R}_0^+$, which is continuous in $\frac{v}{\|v\|}, \frac{w}{\|w\|}$, convex in A and for a constant $m < q$ the estimate $g(v, w, A) - g(u, w, A) \leq C_g\, \|v - u\|\, (1 + \|A\|_2^m)$ holds for all $u, v, w \in S^2$ and $A \in \mathbb{R}^{3,3}$. Then $E[\cdot]$ attains its minimum over all deformations $\phi \in \mathcal{A}$ and the minimizing deformation ϕ is a homeomorphism and in particular $\det D\phi > 0$ *a.e. in* Ω.

As a useful class of examples let us examine the following functions.

- $g(v, w, A) := 1 - |v \cdot Aw|^\gamma$, with $\gamma \geq 1$.
- $g(v, w, A) := \|(1\!\!1 - v \otimes v) \cdot Aw\|^\gamma$, with $\gamma \geq 1$. Recall that $1\!\!1 - v \otimes v = (\delta_{ij} - v_i\, v_j)_{ij}$ is the projection onto the plane normal to v, provided $\|v\| = 1$.

4. REGULARIZATION

The morphological energies introduced are invariant for possibly very irregular modifications of a minimizing deformation ϕ. This indicates, that the set of minimizers is very irregular and hence, minimizing solely the matching energy is an ill-posed problem. As a regularization, we interprete Ω as an isotropic elastic body and incorporate an additional hyperelastic energy E_{reg},

$$E[\phi] = E_m[\phi] + E_{reg}[\phi]\,, \qquad (4)$$

which controls length, area and volume deformations separately:

$$\begin{aligned}E_{reg}[\phi] := \int_\Omega\ & a||D\phi||_2^p && \text{(length control)}\\ +\ & b||\mathrm{Cof}\, D\phi||_2^q && \text{(area control)}\\ +\ & \Gamma(\det D\phi)\, \mathrm{d}\mu && \text{(volume control)},\end{aligned}$$

with $\Gamma(D) \to \infty$ for $D \to 0, \infty$, e. g., $\Gamma(D) = \gamma D^2 - \delta \ln D$. Let us remark, that we want to ensure certain essential properties of the minimizing deformation, especially that no interpenetration of matter occurs, which would be impossible to rule out with linear elastic models. Here, the matching energy may be interpreted as contributing external forces to an elastic modeling problem.

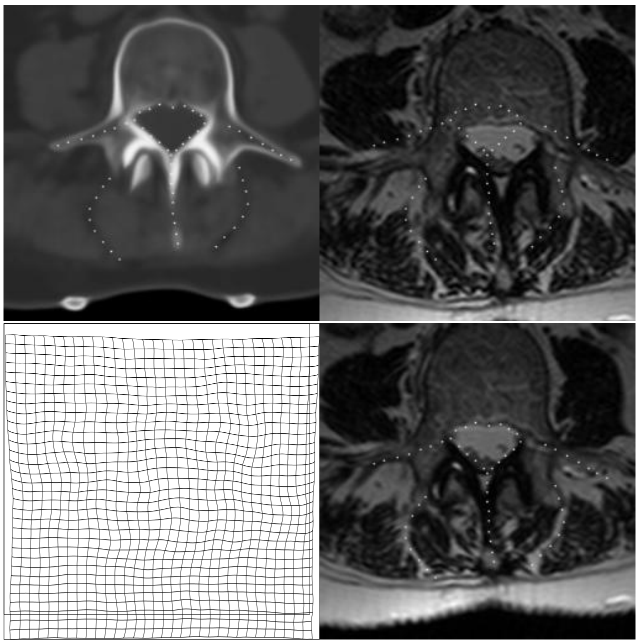

Fig. 2. Axial MR-CT registration of a human spine. Dotted lines mark certain features visible in the reference image. They are repeatedly drawn at the same position in the other images. *Top Left:* reference, CT, *Top Right:* template, MR. *Bottom Left:* deformation plot of registration. *Bottom Right:* deformed template $T \circ \phi$ after final registration. Results have been improved by an additional feature energy $\int_\Omega |d(\phi(\cdot), \mathcal{F}_T) - d(\cdot, \mathcal{F}_R)|^2 \, \mathrm{d}\mu$, where d is a weighted distance function and the "feature sets" $\mathcal{F}_T$, $\mathcal{F}_R$ here roughly mark the boundary of the body (bottom) and the upper round part of the vertebra's border.

5. MINIMIZATION OF THE ENERGY

We want to minimize the combined energy by a classical descent method, i. e.,

$$\partial_t \phi = d(E'[\phi])$$

where $(d(E'[\phi]), E'[\phi]) \leq 0$, which means that d produces a suitable descent direction. Examining real images, we expect the energy landscape to be very irregular and since we are furthermore extracting the normal fields of the images, we have to be careful whenever we compute derivatives. In what follows, we point out some important numerical issues, details are however beyond the scope of this summary.

1. If one starts the minimization directly on the finest scale of resolution, a stable computation can of course not be expected. Representing the input data N_T, N_R in a convenient **morphological scale space** allows to begin the minimization of a coarse scale, where the energy landscape is very regular and eventually offers uniqueness and successively continue the computation by stepping over to finer scales. We can select from a wide range of well-known filters on the images itself or alternatively work on the Gauss maps themselves, privilegeing directions at locations, where the image shows significant geometrical features. In the paper we would like to present such techniques. Here, also the "morphology" from a clinical viewpoint may be taken into account.

2. Since the morphological energies are depending on the normals of the input images and in the vicinity of significant edges the normals can be expected to be almost constant in the perpendicular direction, a displacement of edges is hardly detected by the energy. The minimization rather focuses on correct alignment. Often, the descent direction is given by the L^2-representation of the Frechét-derivative of E, given by

   $$(\mathrm{grad} E, \theta) = \langle E'[u], \theta \rangle.$$

   But we may favor certain properties of the descent direction, according to the intrinsic structure of the formulated problem. We may for example in the early stage of the minimization privilege translations, i. e., low frequency updates to the deformation ϕ. To this end, given a metric g, we chose g-representations of the gradient

   $$g(\mathrm{grad}_g E, \theta) = \langle E'[u], \theta \rangle,$$

   which hence can be able to control such properties for special choices of metrics.

3. The nonlinear elastic energy E_{reg} is, as it is formulated above is homogenous and isotropic. Here, also the morphological structure may be exploited to enhance for example the stability of curvilinear structures or other material properties that can be derived directly from the geometry of the images.

4. The problem is discretized using multilinear Finite Element spaces using a nested multilevel hierarchy for the representation of the input and deformation on different levels with a coupling to the scale parameter.

6. CONCLUSION

The novel geometric approach presented here is especially characterized by the invariance to not necessarily monotone locally invertible contrast transformations. It's design allows a good interpretation of the registration, since it is based on simple geometric entities and is locally able to recover even non-rigid misalignments of the data. It is thus suitable for the registration of multi modal data, as confirmed by some numerical results. Unlike for several other registration techniques, both the theory as well as practical issues are developed. We think that this model bears a wide range of potential for further improvement.

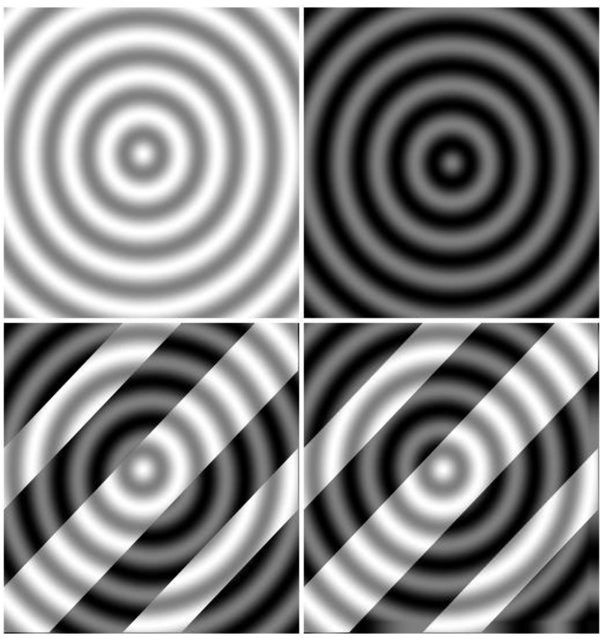

Fig. 3. An artificial test example: concentric, radial isolines. *Top row:* reference (left) and shifted template image with contrast change (right). *Bottom row:* Initial misfit (left) and registration result (right), computed over several scales.

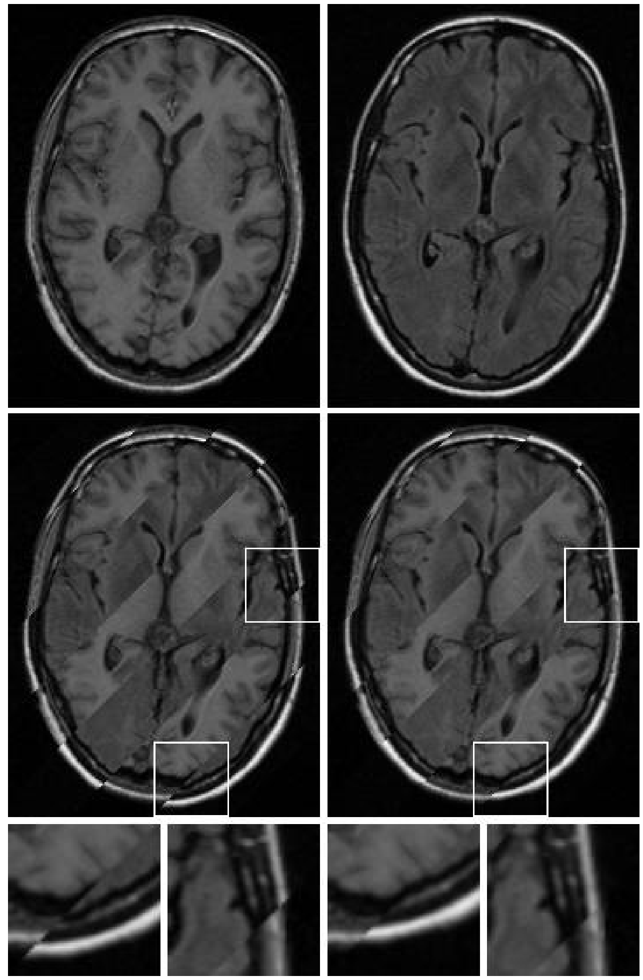

Fig. 4. Registration results for a pair of real data sets. *Top left:* reference image, modality: MR. *Top right:* template image, modality: FLAIR. *Middle left:* misfit before the registration. *Middle right:* registration result. *Bottom row:* zoom-up of the areas marked in the middle row images.

7. REFERENCES

[1] J. BALL, *Global invertibility of Sobolev functions and the interpenetration of matter*, Proc. Roy. Soc. Edinburgh, 88A (1988), pp. 315–328.

[2] G. E. CHRISTENSEN, R. D. RABBITT, AND M. I. MILLER, *Deformable templates using large deformation kinematics*, IEEE Trans. Medical Imaging, 5, no. 10 (1996), pp. 1435–1447.

[3] P. G. CIARLET, *Three-Dimensional Elasticity*, Elsevier, New York, 1988.

[4] U. CLARENZ, M. DROSKE, AND M. RUMPF, *Towards fast non–rigid registration*, in Inverse Problems, Image Analysis and Medical Imaging, AMS, 2001.

[5] M. DROSKE AND M. RUMPF, *A variational approach to non-rigid morphological registration*, SIAM Appl. Math., (2003). submitted.

[6] U. GRENANDER AND M. I. MILLER, *Computational anatomy: An emerging discipline*, Quarterly Appl. Math., LVI, no. 4 (1998), pp. 617–694.

[7] G. SAPIRO, *Geometric Partial Differential Equations and Image Analysis*, Cambridge University Press, 2001.