PREDICTING THE PERMEABILITY OF TEXTILE REINFORCEMENTS VIA A HYBRID NAVIER-STOKES/BRINKMAN SOLVER

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ABSTRACT: Numerical computation of textile permeability is important for composite manufacturing. Using Darcy's law, permeability can be derived from a simulation of the fluid flow, i.e. after solving the Stokes, Navier-Stokes or Brinkman equations. The latter allow to model intra-yarn flow in case of permeable yarns. In this paper we present a numerical method for the calculation of the permeability of textile models based on a finite difference discretisation of the partial differential equations. Two different formulas for the calculation of the local permeability are discussed. Theoretical, numerical and in particular experimental validation is presented.

KEYWORDS: Textile composites, Permeability, CFD, Homogenisation

INTRODUCTION

For the manufacturing of composites with textile reinforcement, the permeability of the textile is a key characteristic and is of particular importance for the injection stage of Liquid Composite Moulding. The prediction of textile permeability is important due to the often encountered problems of non-uniform impregnation, which may even involve void and dry spot formation. Permeability is a geometric characteristic related to the structural features of the textile at several length scales. Textiles are porous media and the permeability tensor can be defined by Darcy's law

$$\left\langle \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \right\rangle = -Re \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \left\langle \begin{pmatrix} \dfrac{\partial P}{\partial x} \\ \dfrac{\partial P}{\partial y} \\ \dfrac{\partial P}{\partial z} \end{pmatrix} \right\rangle . \tag{1}$$

Here, Re denotes the Reynolds number, $\vec{u} = \vec{u}(x, y, z)$ the fluid velocity, $P = P(x, y, z)$ the pressure, $\underline{\underline{K}}$ the permeability tensor of the porous medium and $\langle \ \rangle$ denotes volume averaging. Eqn. 1 is a homogenized equation, where information about the internal geometry of the reinforcement is taken into account in $\underline{\underline{K}}$. Finite element or finite difference Darcy solvers thus require $\underline{\underline{K}}$ as input. Since measurements of textile permeability are time- and resource-consuming, a reliable numerical prediction of $\underline{\underline{K}}$ is required.

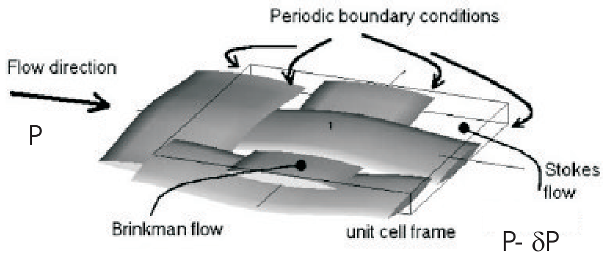

Fig. 1 A unit cell setup

For the calculation of $\underline{\underline{K}}$, we simulate the flow in a unit cell (Fig. 1) since textile has a periodic pattern. As textiles are also hierarchically structured materials, our model for fluid flow must also take into consideration the possible porosity of the yarns. Hence, in the following, if the yarns are porous, we will differentiate between inter-yarn flow and intra-yarn flow. The porosity is accounted for by the permeability tensor K_{tow}. In both cases we aim at the computation of the fluid velocity $\vec{u}$ and the pressure P in order to solve Darcy's law (1) for $\underline{\underline{K}}$.

In the case that the model is limited to creeping, single-phase, isothermal, unidirectional saturated flow of a Newtonian fluid, the inter-yarn flow is described by the incompressible Navier-Stokes equations (here in dimensionless form),

$$\begin{cases} \dfrac{\delta \vec{u}}{\delta t} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla P + \dfrac{1}{Re}\Delta \vec{u} \\ \nabla \cdot \vec{u} = 0 \end{cases}. \tag{2}$$

Here, $\vec{u} = \vec{u}(x, y, z, t)$ and $P = P(x, y, z, t)$. If Re is small, the convective term can be neglected, and Eqn. 2 result in the Stokes equations. Later in this paper, we show numerically that for our applications both the Navier-Stokes and the Stokes equations can be used.

Intra-yarn flow depends on the local permeability tensor K_{tow} of the tow, and is described by the Brinkman equations [5] satisfying

$$\begin{cases} \dfrac{\delta \vec{u}}{\delta t} + (\vec{u} \cdot \nabla)\vec{u} + \dfrac{1}{Re}\underline{\underline{K_{tow}^{-1}}} \cdot \vec{u} = -\nabla P + \dfrac{1}{Re}\Delta \vec{u} \\ \nabla \cdot \vec{u} = 0 \end{cases}, \tag{3}$$

with the convection term included.

We develop numerical software for the calculation of the permeability of textiles, named *FlowTex*. The input of a single layer of the textile model is provided by the *WiseTex* software [11,13,19] which allows the characterisation of a single-layer of the reinforcement or a regularly or randomly nested laminate [12].

NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS

For flow simulations in the irregular geometry of a textile, we have chosen to solve Eqn. (2) numerically on a regular staggered grid with a finite difference discretisation. An example of a textile geometry and its discretisation on a regular grid is shown in Fig. 2.

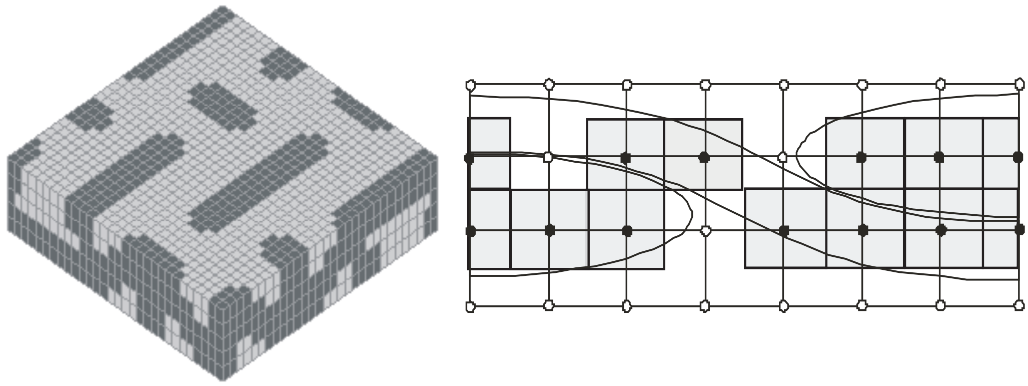

Fig 2 A 3D and 2D voxel representation of a textile geometry

In previous work the solution was performed using lattice Boltzmann algorithm [3]. The implementation described in this paper is based on the 3D finite difference Navier-Stokes solver *NaSt3DGP*, developed at the *Institute for Numerical Simulation* of the University of Bonn [1,7]. In order to apply the code for the computation of the permeability of textiles, several extensions to the code have been made. An interface between *FlowTex* and *NaSt3DGP* allows the input of the voxel description of the textile geometry [13,19] provided by WiseTex (Fig. 2). For the unit cell setup, we implemented periodic boundary conditions in three directions for the velocity, and periodic boundary conditions up to a constant gradient for the pressure (Fig. 1). To account for intra-yarn flow, the code has been extended to solve the Brinkman equations: we solve Eqn. 3 on the whole domain with variable $0 < K_{tow} \leq \infty$. It was shown by Angot [2] that this is a valid approach in which no extra interface conditions between the fluid and the porous part are required. This approach is a practical method to deal with the coupled problem of flow in a porous medium and flow in-between the yarns. An implicit treatment of the diffusive terms for the Navier-Stokes/Brinkman equations has substantially improved the speed of the permeability computations [17].

ANALYTICAL VALIDATION

In our previous papers [17,18] we presented numerical and experimental results that take the textile geometry at three different length scales into account: the macroscale of the textile part, the mesoscopic scale of the unit repeat cell, as well as the microscopic scale of the fibres within the yarns. In this section we give an analytical validation of the proposed Darcy's law for the calculation of the inter-yarn permeability with the help of homogenisation theory. Also, homogenisation theory provides us with a new possibility for the computation of the permeability tensor. Numerical validation of the latter approach will be given further in this paper.

The direct numerical treatment of fluid flow in porous media is difficult and time consuming due to the rapid variations of the pore scale. However, when the characteristic size of the obstacles in a repeat cell of the medium, e.g. of the yarns, is small compared to the whole sample, homogenisation theory allows us to "average" or "upscale" the equations of fluid mechanics that hold on one scale of the porous medium to the next scales. Hence, we avoid the solution of the fluid equations in the complicated pore geometry by merely studying the geometry's homogenised influence on these equations [14].

Several authors have dealt with the homogenisation of the Stokes or Navier-Stokes equations in a periodic porous medium [9,14,15] and derive Darcy's law as the limiting equation in the homogenisation process. In Darcy's law information about the structure of the pore scale is only kept through the effective quantity of permeability. The permeability tensor $\underline{\underline{K}}$ in homogenisation theory is given as $K_{ij} = \frac{1}{|Y|}\int_Y \vec{w}_j^i \,\mathrm{d}\tau$, where j denotes the j-th component of the vectors w^i, which for $1 \le i \le n$ are the solutions of the so-called "cell problems":

$$\begin{aligned} &-\Delta_\tau \vec{w}^i + \nabla_\tau \pi^i = \vec{e}^i \text{ in } Y_F \\ &\mathrm{div}_\tau\, \vec{w}^i = 0 \text{ in } Y_F \\ &\vec{w}^i = 0 \text{ on } \partial Y_S \text{ and } \left\{\vec{w}^i, \pi^i\right\} \text{ is } Y-\text{periodic} \end{aligned} \tag{4}$$

Here, $\vec{e}^i$ denotes the unit vector, $\tau \in Y$, Y the unit repeat cell of the porous medium and Y_F and Y_S its corresponding fluid and solid part. Furthermore, $\vec{w}(\tau)$ and $\pi(\tau)$ are comparable to the fluid velocity and pressure of the Stokes equation.

From a numerical point of view, this offers a further possibility for the computation of the permeability tensor. The solution of the above cell problems in 3D amounts to three Stokes equations with external forces $(\vec{e}_i)_{1\le i\le 3}$, from which we obtain $\left(\vec{w}^i\right)_{1\le i\le 3}$ for the input of $\underline{\underline{K}}$. This leads to the same results as the computation of $\underline{\underline{K}}$ by Darcy's law since in the unit repeat cell these are equivalent problems [14]. Note that this method gives a straightforward definition for the computation of all components of $\underline{\underline{K}}$ whereas the calculation of $\underline{\underline{K}}$ via Darcy's law requires solving a 9x9 system of equations. However, for the calculation of e.g. K_{xx}, we neglect the influence of K_{xy} and K_{xz} in (1) which according to Table 1 is allowed and yields a direct calculation of K_{xx}.

LOCAL PERMEABILITY

If we want to include the intra-yarn flow into the flow simulations, we solve the Brinkman equations (3), which requires the local permeability in every grid point which lies inside the yarn. At micro-level, the fibres are considered as regularly packed cylinders. Gebart [6] presents analytical formulas for the permeability of a porous medium which consists of a quadratic packing of cylinders for both flow along and transversal to the cylinders

$$K_{Gebart,Along} = \frac{8}{57}\frac{\left(1-V_f\right)^3}{V_f^{\,2}} r^2, \tag{5}$$

$$K_{Gebart,Trans} = \frac{16}{9\pi\sqrt{2}}\left(\sqrt{\frac{V_{f\,\max}}{V_f}}-1\right)^{2.5} r^2, \tag{6}$$

with V_f the local volume fraction, r the radius of the cylinders and $V_{f\,\max} = \pi/4$. Berdichevsky et al. [4] on the other hand present formulas for the local permeability

$$K_{Berdi,Along} = \frac{r^2}{8V_f}\left(\ln\frac{1}{V_f^{\,2}} - \left(3-V_f\right)\left(1-V_f\right)\right) \tag{7}$$

$$K_{Berdi,Trans} = \frac{r^2}{8V_f}\left(\ln\frac{1}{V_f} - \frac{\left(1-V_f\right)^2}{\left(1+V_f\right)^2}\right). \tag{8}$$

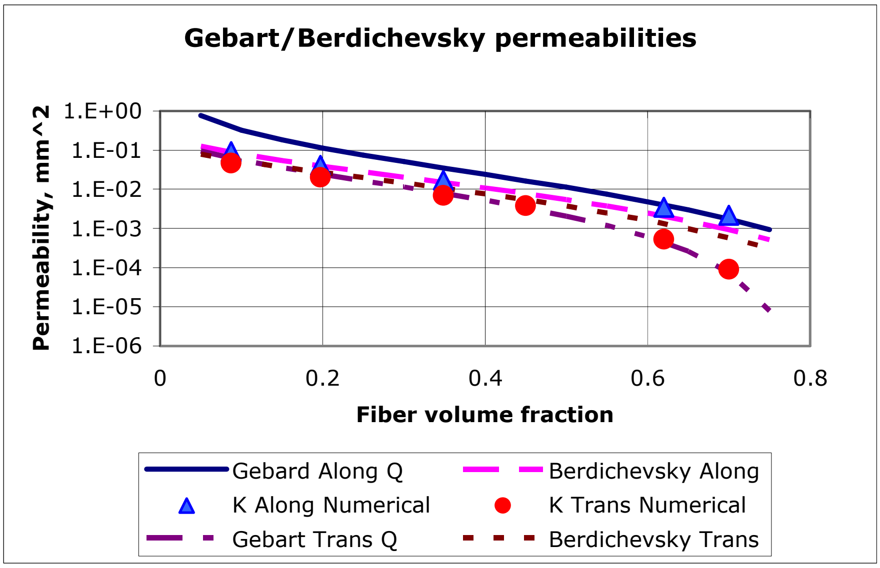

Fig. 2 Comparison of the Gebart/Berdichevsky formulas with numerical results

Fig. 2 shows a plot of the different formulas. We see that for the permeability along the fibres, the curves of Gebart and Berdichevsky show comparable results, although the formulas of Gebart give a higher permeability. For the permeability in the transversal direction, however, the formulas give different results for higher volume fractions.

Fig. 2 also shows the results of our computations with the software described above for a parallel square array of cylinders. The formula of Berdichevsky matches better with the numerical results for the flow along the fibres, although not for higher volume fractions. However, for the flow in the transversal direction, clearly the formula of Gebart gives better results. The *FlowTex* software calculates the local permeability in (and transversal to) the direction of the fibres according to (7) and (6). Once K_A and K_T are known, they are projected onto the main directions X, Y, Z which then yields the local permeability tensor K_{tow}.

VALIDATION

Analytical data

In this section we compare the numerical results of the permeability of a cubic array of spheres with analytical results. On the one hand we can compute the flow field with the Stokes equations and obtain $\underline{\underline{K}}$ from the applied pressure drop ∇P and the average velocity field $\langle \vec{u} \rangle$ in Darcy's law (1) and on the other hand the same permeability will be obtained by solving the cell problems (4). For a periodic array of spheres Sangani and Acrivos [16] found general solutions of the Stokes equations in series formulation, whose coefficients are determined numerically. For several volume fractions Vf the authors computed the dimensionless drag force F to which the first entry of the permeability tensor is related by $K_{xx} = 1/6\pi rF$, with r the sphere radius.
Both the permeabilities from numerical simulations as well as the semi-analytical ones are listed in Table 1 for various values of $\chi = \left(Vf / Vf_{\max}\right)^{1/3}$, which is a scaled sphere volume fraction $Vf = 4\pi r^3 / 3L^3$, where $Vf_{\max} = \pi/6$ corresponds to the case where the spheres are in contact. First of all, we note that all the values are in good agreement with those obtained analytically by Sangani and Acrivos [16] and deviate no more than 0.5% from them. Furthermore, the results obtained from the cell problem and by Darcy's law are equal. This was to be expected as in homogenisation theory the cell problem is just an auxiliary problem for the definition of the permeability tensor and the derivation of Darcy's law. But also for actual text geometries, the permeability tensor obtained from the cell problem is accurate as shown in Table 2. Hence, homogenisation theory not only applies to our textiles but also offers an easier way to implement an efficient method for permeability computations.

Table 1: Computation of the permeability K_{xx} for a simple cubic array of spheres

χ	Resolution	K_{xx} : Darcy's Law	K_{xx} : Cell Problem	K_{xx} : Analytical
0.2	60^3	3.8135 E-01	3.8135 E-01	3.8129 E-01
0.4	60^3	1.2314E-01	1.2314E-01	1.2327E-01
0.8	80^3	1.3118E-02	1.3118E-02	1.3197E-02
1	100^3	2.5083E-03	2.5083E-03	2.5203E-03

Experimental validation

A comparison between the results of the Navier-Stokes/Brinkman solver with experimental data is given in Table 2. Information on the *Natte* textile and the *Carbon woven fabric* can be found in [8,10]. The Numerical and experimental results are in good agreement for the *Natte* textile (Fig. 3) and give reasonable results for the *Carbon woven fabric*. Table 2 also shows the results for a Parallel Square Array (PSA) of cylinders.
For a typical unit cell of textile, with a flow velocity typically used in Resin Transfer Moulding ($10^{-3} m/s$), the Reynolds number is about 0.05. As flows with a low Reynolds number can be described by the Stokes equations, we compare the solution of the Navier-Stokes equations with the solution of the Stokes equations (Table 2).
This shows that for our application, we do not have to solve the non-linear Navier-Stokes equations with pseudo time-stepping, but can solve the steady Stokes equations with a preconditioned iterative solver instead. This can lead to a considerable speedup of

the permeability simulations in comparison with the time-stepping we use now to solve the Stokes equations. Such a sophisticated Stokes solver is presently under development.

Table 2 Comparison between Navier-Stokes and Stokes calculations

Method \ Setup	PSA V_f 62%	Natte	Carbon woven fabric
K_{xx} Navier-Stokes *(mm²)*	3.4e-03	3.3e-04	4.2e-04
K_{xx} Stokes *(mm²)*	3.4e-03	3.3e-04	4.2e-04
K_{xx} Cell Problem *(mm²)*	3.4e-03	3.3e-04	4.2e-04
K_{xx} Experimental *(mm²)*	-	2.7e-04 ±10%	1.0e-04 ±10%

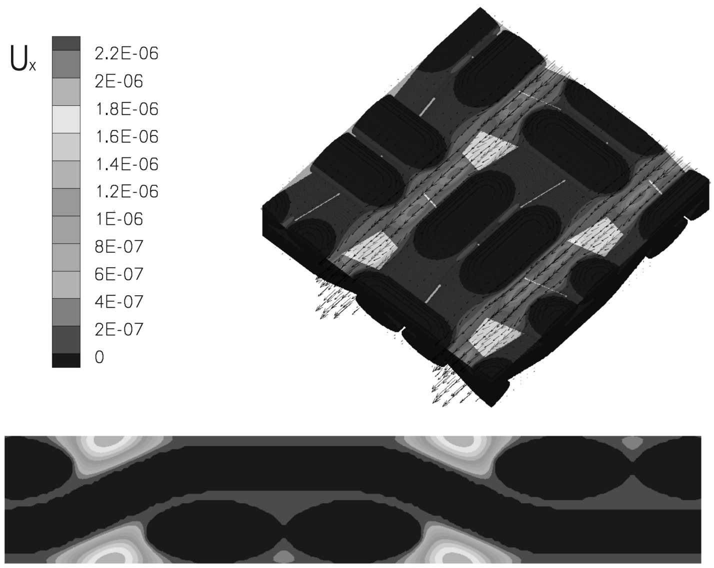

Fig. 3. 3D image and a 2D cut of the calculated flow field in the Natte model

CONCLUSIONS

Two methods for the calculation of the permeability of textiles have been presented. The solution of the Navier-Stokes/Brinkman equations with a finite difference solver yields the velocity and pressure field for Darcy's law. On the other hand, the permeability can be calculated via the definitions given by the theory of homogenisation. Both methods lead to the same numerical results, hence solving the Navier-Stokes/Brinkman equations on the unit-cell is a correct approach to obtain the textile permeability. Furthermore, the numerical results are in good agreement with experimental results.
Two formulas for the local permeability term of the Brinkman equation were discussed and compared with numerical results.

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REFERENCES

1. http://wissrech.iam.uni-bonn.de/research/projects/nast3dgp
2. P. Angot, "Analysis of Singular Perturbations on the Brinkman Problem for Fictitious Domain Models of Viscous Flows", *Mathematical Methods in the Applied Sciences,* Vol. 22, 1999 pp. 1395.
3. E.B. Belov, S.V. Lomov, I. Verpoest, ea, "Modelling of permeability of textile reinforcements: Lattice Boltzmann method", *Composites Science and Technology*, Vol. 64, 2004, pp. 1069.
4. A. Berdichevsky and Z. Cai, "Preform Permeability Predictions by Self-Consistent Method and Finit Element Simulation", *Polymer Composites*, Vol. 14, 1993 pp. 132.
5. H.C. Brinkman, "On the permeability of media consisting of closely packed porous particles", *Applied Scientific Research*, Vol. 1, no. 1, 1947 pp.81.
6. B. Gebart, "Permeability of Unidirectional Reinforcements for RTM", *Journal of Composite Materials*, Vol. 26, no. 8, 1992 pp. 1100.
7. M. Griebel, T. Dornseifer and T. Neunhoeffer, "Numerical Simulation in Fluid Dynamics, a Practical Introduction", SIAM, Philadelphia, 1998.
8. K. Hoes, H. Sol, D. Dinescu, ea., "Study of nesting induced scatter of permeability values in layered reinforcement fabrics", *Composites part A*, Vol. 35, 2004, pp.1407.
9. U. Hornung (Ed.), "Homogenization and Porous Media", Springer Verlag, 1997.
10. B. Laine, G. Hivet, P. Boisse, ea, "Permeability of the woven fabrics: a parametric study", *Proc. of the 8th ESAFORM Conf. on Material Forming*, 2005. pp. 995
11. S.V. Lomov, G. Huysmans, Y. Luo, ea., "Textile composites models: Integrating strategies", *Composites Part A*, Vol. 32, no. 10, 2001 pp. 1379.
12. S.V. Lomov, I. Verpoest, T. Peeters, ea., "Nesting in textile laminates: Geometrical modelling of the laminate", *Composites Science and Technology*, Vol. 63, 2002 pp. 993.
13. S.V. Lomov, I. Verpoest, B. Verleye ea., "WiseTex based models of permeability of textiles", *Proc. of the 8th Int. Conf. on Flow Processes in Composite Materials.*
14. A. Mikelic, "Homogenization theory and applications to filtration through porous media", *Filtration in porous media and industrial application*, Lecture Notes in Mathematics, Vol. 1734, 2000, pp. 127.
15. E. Sanchez-Palencia, "Non-homogeneous media and vibration theory", *Lecture notes in Phys.*, Springer Verlag, 1980 pp. 127.
16. A. S. Sangani and A. Acrivos, "Slow flow through a periodic array of spheres", *Int. J. Multiphase Flow*, Vol. 8, no. 4, 1982 pp. 343.
17. B. Verleye, M. Klitz, R. Croce, ea., "Computation of the permeability of texiles", Report TW 455, Dept. Computer Science, K.U.Leuven, Leuven, Belgium, April 2006.
18. B. Verleye, M. Klitz, R. Croce, ea., "Computation of Permeability of Textile Reinforcements", *Proc. of IMACS 2005,* CD-edition, 2005.
19. I. Verpoest and S.V. Lomov "Virtual textile composites software Wisetex: integration with micro-mechanical, permeability and structural analysis." *Composites Science and Technology*, Vol. 65, 2005, pp. 2563.