A Procedural Interface for Multiresolutional Visualization of General Numerical Data

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Abstract

Together with a rapid development of computer hardware, sophisticated, efficient numerical algorithms allow simulation computations of complex physical phenomena. Methods, such as Finite Volume, Multigrid Finite Element schemes, Sparse Grid, Wavelet approaches, and Particle Methods or Gridless Discretizations all carry their own, tailored data structures, which reflect the decomposition of the function spaces as well as the decomposition in physical space.

Multiresolutional visualization on numerical data is described as an indispensable ingredient of real time interactive post processing. The typically enormous data bases are locally resolved on different levels of detail to achieve a significant saving of CPU and rendering time.

For efficient data analysis and graphical post processing the method of spatial, hierarchical subdivision combined with the recovery of the local function spaces is presented. To manage a variety of different numerical data a general procedural interface to arbitrary large numerical data sets is presented. This leads to a visualization beyond prescribed data formats. Discrete numerical solution data is directly addressed in the user's data structures. Furthermore the procedural interface supports a flexible method of local error measurement, again encapsulated in certain user supplied functions. The software conception, its data classes and methods are described and the setup of the corresponding procedural user interfaces is discussed in detail.

Examples from various numerical methods and different data bases underline the applicability of the proposed concept.

Contents

1 Description of the Interface Structures

The base structure implementing the multiresolution concepts is HMesh3d and HMesh2d for three and two space dimensions respectively. For the description of the implementational aspects we will always restrict to either the three dimesional or the two dimensional case, as the concepts for both situations are transferable. The hierarchical concept (HMesh3d) is implemented as a subclass of Mesh3d. Additionally to Mesh3d, which deals with non–hierarchical meshes, (HMesh3d) is enlarged by the hierarchical element functionality and expanded interface structures. Although MESH3D has already been described in an earlier paper [34], its documentation is in revised form listed once more for the sake of completeness and with respect to actualizations.

Hierarchical Element

In this mesh concept we use a procedural access to single elements. The procedures return element data in a small structure, these procedures are described later on. One example is for instance

```
HELEMENT3D *first_macro(HMESH3D *mesh, MESH_ELEMENT_FLAGS required);
```

This — user supplied — function has to return the first macro element of a given HMesh3d. The flags required specify, which parts of the HELEMENT3D–structure are to be filled at least.

```
typedef struct helement3d
{
  MESH3D                 *mesh;
  double                 **vertex;
  int                    *vindex;
  int                    eindex;
  ELEMENT3D_DESCRIPTION  *descr;
  MESH_ELEMENT_FLAGS     present;
  void                   *user_data;
  HELEMENT3D             *parent;
  VINHERIT               *vinh;
  int                    ref_rule;
  int                    level;

} HELEMENT3D;
```

The structure for an element of the mesh consists mainly of a pointer descr to an element description, which contains all information described above, and a vector of pointers vertex to the world coordinates of the element's vertices.
Such information may be enlarged by optional global eindex and vindex[] integer indices of the element itself and its vertices and a pointer to a data area which contains additional information that may be used by the transformation routines or any other of the user-supplied routines.
The vertex indices, if present, allow the visualization routines to minimize the number of calls to the data function, if the data values are globally continuous. In this case, the values at a node are uniquely defined, independent of the element where it belongs to. If the data is not continuous between elements, the global vertex index may also be used to compute a continuous approximation to the data by averaging the values at the nodes.
The global element indices for example may be used to specify a set of elements of special attention and to display only these elements (or their neighbourhood).
The flag variable present shows, which data arrays are set, at least the ones, which were set in the function call. The pointer parent points to the HELEMENT-structure of the parent element, if there is one. This allows a flexible handling of the existing list of element, from the coarsest parent element to the current decendent element.
Each of the later described funtions has to set the mesh, descr, present and parent entries. Since mesh is an HMesh in this context is must of course be downcasted. If the max_eindex entry of mesh is nonzero, eindex must be set to the element's global number, else to zero. ref_rule is completely optional. The optional user_data

can point to additional data and is not dereferenced in the library. `vertex`, `vindex` and `vinh` only need to be set, if the `required` flags of the function call says so. `vertex` points to an array of pointers, pointing to an array containing the world coordinates of the vertices. If the `max_vindex` entry of `mesh` is nonzero, `vindex` must point to an array of the vertice's global numbers, else it must be `NULL`. `vinh` points to an array of `VINHERIT` structures
for each vertex, or is `NULL`.
`descr` points to an element description, which is constant for a whole class of elements, and which will be described hereafter:

The geometry of a single element is described using a polygon–oriented boundary representation of the parameter domain. As we support only (curved) polyhedral elements, an element can be described by the set of its vertices and a description of the boundary polygons. This applies only to the local coordinates of an element; its shape in world coordinates is determined by a transformation routine.
For a three dimensional polyhedron, we specify the number of boundary polygons (faces) and for each of these polygons the number of vertices, the local vertex indices and their order (thus giving an orientation to the polygon such that the surface normal is the inner normal to the polyhedron), and the local indices of the adjacent face across each of the edges of the polygon.
For each vertex, the coordinates in the local coordinate system of the element are given by the element description. Based on these values, the visualization routines may operate in the local coordinate space. The pointer to a function `check_inside()` is provided by the element description, which checks whether a point in local coordinates is inside the element or not.
The element description is completed by pointers to transformation routines from local coordinates to world coordinates and vice versa and routines `boundary()` and `neighbour()`, which give information about the neighbourhood of elements. Additionally, the transformation routine from world to local coordinates checks whether a given point is inside the element or not.

```
typedef struct helement3d_description
{
  int         dindex;
  int         number_of_vertices;
  int         number_of_polygons;
  int         *polygon_length;
  int         **polygon_vertex;
  int         **polygon_neighbour;
  int         dimension_of_coord;
  double      **coord;
  int         parametric_degree;
  int         (*world_to_coord)(HELEMENT3D *, double *, double *);
  void        (*coord_to_world)(HELEMENT3D *, double *, double *);
  int         (*check_inside)  (HELEMENT3D *, double *);

  HELEMENT3D  *(*neighbour)(HELEMENT3D *, int, int, double *, double *, MESH_ELEMENT_FLAGS);
  int         (*boundary)  (HELEMENT3D *,  int);

  void        (*get_boundary_vertex_estimate)(HELEMENT3D *, double *, int );
  double      (*get_boundary_face_estimate)  (HELEMENT3D *, int );
  void        (*coord_of_parent)             (HELEMENT3D *, double *, double *);

} HELEMENT3D_DESCRIPTION;
```

The entries `dindex` to `boundary` are inherited from `HMesh3d`. The variables give following information:

`dindex` is an index of the current description. This is to distinguish descriptions, which might be processed in a distributed environment.

`number_of_vertices` is the number of vertices in an element of this type.

`number_of_polygons` is the number of polygonal faces of an element of this type.

`polygon_length[`j`]` is the count of vertices of the j–th face ($0 \leq j <$ `number_of_polygons`).

`polygon_vertex[`j`][`i`]` is the (local) index (with range $[0, \ldots,$ `number_of_vertices` $- 1]$) of the i–th vertex from the j–th face ($0 \leq j <$ `number_of_polygons`, $0 \leq i <$ `polygon_length[`j`]`).

`polygon_neighbour[`j`][`i`]` is the index of the adjacent face to the j–th face across the edge from vertex `polygon_vertex[`j`][`i`]` to vertex `polygon_vertex[`j`][`$i+1$`]` ($0 \leq j <$ `number_of_polygons`, $0 \leq i <$ `polygon_length[`j`]`).

`dimension_of_coord` is the dimension of the local coordinates.

`coord[`k`][`i`]` is the i–th local coordinate of the k–th vertex from the polyhedron ($0 \leq i <$ `dimension_of_coord`, $0 \leq k <$ `number_of_vertices`).

`parametric_degree` is the degree of parametric elements.

`check_inside(element3d, coord)` returns `INSIDE` (-1) if the point with local coordinates `coord` is inside of the element's parameter range. Otherwise, at least one plane of a local face separates the element and the given point. In this case the local index of such a face is returned.

`world_to_coord(element3d, xyz, coord)` transforms a point from given world coordinates `xyz[3]` to `coord[dimension_of_coords]` in the local coordinate system of `element3d`. The return value is identical to that from `check_inside()`.

`coord_to_world(element3d, coord, xyz)` transforms a point from local coordinates `coord[dimension_of_coords]` to world coordinates `xyz[3]`.

`neighbour(element3d, polygon, flag, coord, xyz, required)` returns a pointer to an adjacent element of `element3d` across face number `polygon`. Depending on `flag`, the routine returns the first neighbour or a next one (in case of non–conforming meshes the adjacency may be not one to one). If `coord` is not `NULL`, the routine transforms those coordinates from the old local coordinate system to the new one. In many cases this transformation can be supported by the optional parameter `xyz`, which provides world coordinates of the point if available, otherwise this parameter has to be `NULL`. The structure `element3d` may be overwritten by the routine. On hierarchical data not explicitely storing neighbour pointers, it is a bad idea to implement this function, set its pointer to `NULL` to mark this.

`boundary(element3d, polygon)` returns 0, if `polygon` is an inner face of the mesh, a negative integer, if `polygon` is part of the mesh's outer boundary and a positive integer for interior boundary layers.

`get_boundary_vertex_estimate(element3d, estimate, face)` determines the geometry boundary estimator for vertices lying on face `face` of `element3d`, using the current level.

`get_boundary_face_estimate(element3d, face)` returns the geometry boundary estimator for face `face` of `element3d`, taking into account the finer level.

`coord_of_parent(element3d, child_coord, parent_coord)` determines the local coordinates of a point, given in `element3d`'s local coordinates `child_coord`, in its parent's local coordinates `parent_coord`. Since this can be implemented in a less efficient way using the above funtions, there is a default function for this purpose, named `g_hmesh3d_def_coord_of_parent`.

The number of such element descriptions and therefore the amount of storage for them is equal to the number of different element types in one mesh. There are no copies of this necessary for each element.
In Figure 1 vertex and face relationships for the example of cubes are depicted. `check_inside()` is easily implemented using the local coordinates.

The user's data structures (e.g. from a numerical method) may be completely different to the structures described above. In case of implicitly given Finite Difference meshes, the functions `neighbour` and `boundary` are implemented using simple index arithmetic.

Unstructured grids usually provide such information explicitly.

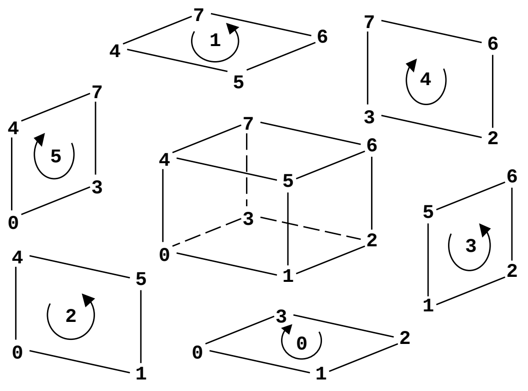


Figure 1: Prototype of a cube.

```
typedef struct vinherit
{
  int     np;
  int     *pindex;
  double  *pweight;

} VINHERIT;
```

This structure defines optional vertex inheritance for a vertex. If the `required` flags order a `vinh` array and none can be given, it has to be set to `NULL`. Ist entries are

`np` number of parent vertices

`pindex` local indices of parent vertices

`pweight` weights for the coordinate calculation

A vertex also appearing in the parent element would have a `VINHERIT` of `{1,{i},{1.0}}`, one being interpolated between two vertices i and j by bisection would have `{2,{i,j},{0.5,0.5}}`.

Supervising Grid Structure

Now we introduce the supervising structures of the actual tree of elements and the function data defined on the corresponding domain. At first we introduce an enumeration type which will serve as a parameter type to describe the required information of a given element for a specific visualization task.

```
typedef enum
{
  hefNone    = 0,
  hefVertex  = 1<<0,
  hefVindex  = 1<<1,
  hefVinh    = 1<<2,
  hefAll     = 1<<0 | 1<<1 | 1<<2

} MESH_ELEMENT_FLAGS;
```

More precise `hefNone` indicates no information except the basic one is to be specified, `hefVertex` marks that vertex coordinates have to be supported. Vertex indices (if present) have to be delivered, if `hefVindex` is set, and finally coordinate inherintence rules have to be supported whenever `hefVinh` is marked.
In what follows the class description for nonhierarchical meshes `MESH3D` and for hierarchical meshes `HMESH3D` are listed.

```
typedef struct mesh3d
{
  int                 max_dimension_of_coord;
  int                 max_eindex;
  int                 max_vindex;
  int                 max_dindex;
  int                 max_number_of_vertices;

  G_LIST              *current_function; /* private */

  MESH_ACCESS_FLAGS   access_mode;
  MESH_ACCESS_FLAGS   access_capability;

  GENMESH_FDATA       *f_data;

  void                *user_data;

  ELEMENT3D           *(*first_element)(MESH3D *,    MESH_ELEMENT_FLAGS);
  ELEMENT3D           *(*next_element) (ELEMENT3D *, MESH_ELEMENT_FLAGS);
  ELEMENT3D           *(*copy_element) (ELEMENT3D *, MESH_ELEMENT_FLAGS);
  void                (*free_element)  (ELEMENT3D *);

  ELEMENT3D           *(*complete_element)(ELEMENT3D *, MESH_ELEMENT_FLAGS);

  int                 (*set_time)(MESH3D *, double t);
  int                 (*get_time)(MESH3D *, double *t, double *tminus, double *tplus);

} MESH3D;
```

This mesh class is a superclass of the following class `hmesh3d`:

```
typedef struct hmesh3d
{
  int                 max_dimension_of_coord;
  int                 max_eindex;
  int                 max_vindex;
  int                 max_dindex;
  int                 max_number_of_vertices;

  G_LIST              *current_function; /* private */

  MESH_ACCESS_FLAGS   access_mode;
  MESH_ACCESS_FLAGS   access_capability;

  GENMESH_FDATA       *f_data;

  void                *user_data;

  ELEMENT3D           *(*first_element)(MESH3D *,    MESH_ELEMENT_FLAGS);
  ELEMENT3D           *(*next_element) (ELEMENT3D *, MESH_ELEMENT_FLAGS);
  ELEMENT3D           *(*copy_element) (ELEMENT3D *, MESH_ELEMENT_FLAGS);
  void                (*free_element)  (ELEMENT3D *);

  ELEMENT3D           *(*complete_element)(ELEMENT3D *, MESH_ELEMENT_FLAGS);
  int               (*set_time)(MESH3D *, double t);
  int               (*get_time)(MESH3D *, double *t, double *tminus, double *tplus);

  HELEMENT3D        *(*first_child) (HELEMENT3D *, MESH_ELEMENT_FLAGS);
  HELEMENT3D        *(*next_child)  (HELEMENT3D *, MESH_ELEMENT_FLAGS);
  HELEMENT3D        *(*first_macro) (HMESH3D *,    MESH_ELEMENT_FLAGS);
  HELEMENT3D        *(*next_macro)  (HELEMENT3D *, MESH_ELEMENT_FLAGS);
  HELEMENT3D        *(*select_child)(HELEMENT3D *, double *, double *, MESH_ELEMENT_FLAGS);
  int               max_level;
  int               level_of_interest;
  double            boundary_threshold;

} HMESH3D;
```

The mesh is just a collection of elements, as they are described in the previous section, together with optional data functions on the mesh. As already mentioned `Mesh3d` only contains the subset of instance variables from `max_dimension_of_coord` to `get_time`.
We restrict the access to elements to a procedural interface, that delivers pointers to elements in one of two possible ways:

- by building a procedurally linked list of elements, that runs sequentially through all elements in the mesh via two procedures

  `ELEMENT *first_element(mesh,required)`,

  `ELEMENT *next_element(element,required)`,

  which return the anchor to the list resp. the next element in the list, having to provide all data not mentioned in `MESH_ELEMENT_FLAGS` and all arrays marked in `required`. As shown later on, for an `HMesh3d` these functions can be implemented using their hierarchical counterparts. Thus default functions are present in new instances.

- we can go from one element to an adjacent element (in space) across the face `polygon` via the routine

  `ELEMENT *neighbour(element, polygon, flag, coord, xyz, required)`

  which is contained in the `HELEMENT3D_DESCRIPTION`.

Both the `next_element()` and `neighbour()` routines may overwrite the `element` data structure such that no additional storage is needed for the next element's data structures. In case, where one needs to collect information about several elements, a routine

`ELEMENT *copy_element(element,required)`

gives a copy of an element, which can be deleted later by

`free_element(element,required)`.

When an element lacks some information, in conformance to its `present` flags, there may be called

`ELEMENT *complete_element(element,required)`

to complete it. Normally it would return the same, updated, element again; of course with correct `present`.
The hierarchical extensions/replacements of `first_element` and `next_element` are

`ELEMENT *first_macro(mesh,required)`,

`ELEMENT *next_macro(element,required)`,

which traverse the macro elements in an equivalent way and

`ELEMENT *first_child(parent,required)`,

```
    ELEMENT *next_child(element,required),
```

where `first_child` returns the first child of `parent` on the next finer level, and subsequent calls of `next_child` traverse the other children on this level.

```
    ELEMENT *select_child(parent,parent_coord,child_coord,required)
```

is only optional and needed for efficient hierarchical searching. If the point with the local coordinates `parent_coord` is contained in a child of `parent`, a new element is created, filled with the child's information, and returned. In addition the local coordinates of the point are converted to the child's local coordinate system and returned in `child_coord`. There is a default function, `g_hmesh3d_def_select_child`, for this using the above described functions, but in many situations there exists a direct access to the children, which results in a speedup.
The variables `access_mode` and `access_capability` indicate the currently set traversal mode and the supported traversal modes respectively. It is the user's responsibility to set the flag *access_capability* and to program the mesh traversal routines (*first_element*, *next_element*, etc.) so that the set access capabilities are indeed supported. At the moment there are only two possible access flags for different types of mesh traversal (c.f. Section 5.6.3 for further explanations:

```
typedef enum {
  mafNone           = 0,
  mafSorted         = 1<<0,
  mafBoundary       = 1<<1
} MESH_ACCESS_FLAGS;
```

The integer `max_dindex` gives an upper bound of the used element description indices.

This procedural access to the mesh elements allows the generation of local element data at the time when it is actually needed. Otherwise, in case of an array or pointered list, complete information for all elements of the mesh would have to be present at the same time (and occupy much more storage).
The routines which generate the element data structures have to convert between the representations via user's data structures and the element structures. They do this mainly by assigning vertex and index information for an element and referring to a previously prepared description. As mentioned above, in case of structured meshes this is done mainly by index arithmetic, whereas for unstructured meshes such information is usually stored with the mesh.
The `MESH3D` data structure contains values for the allocation of temporary memory that some routines may need:

— An upper bound `max_number_of_vertices` for the number of vertices of any element in the mesh such that a routine which wants to store local vertex information knows how much memory it has to allocate.

— Upper bounds `max_eindex` and `max_vindex` for the element index and vertex index, if such information is supplied (then the `eindex` and `vindex[]` entries in the `ELEMENT` data structure deliver values in the range [0, . . ., `max_eindex`−1] resp. [0, . . ., `max_vindex`−1]). If any one of the `max_?index` values is smaller or equal to zero, the elements do not give corresponding index information.

Furthermore the mesh supports function evaluation in an abstract and flexible way. There can be defined several functions on the mesh. They are all independent concerning properties such as continuity and polynomial degree. The entry `current_function` is a pointer to a list of currently supported functions on the mesh. The `GLIST` is a simple list supervising class type.
The functions `set_time()` and `get_time()` enable the handling of timedependent data. Calling `set_time()` with time `t` ensures that the next function evaluation is with respect to this new time. Furthermore, meshes deforming in time, will then – if supported – deliver time dependent coordinate positions. Finally, the access routines consider the correct adaptive grid valid for the specified time, with its uniquely defined tree hierarchy. The call of `get_time()` returns the current time `*t` and the current time interval [`*tminus`,`*tplus`]. In case of adaptive grids changing their grid topology in time, the identification of an time interval is very useful to be sure that the grid is constant on this time interval.
An `HMESH3D` has the additional variables

`max_level`: The maximal hierarchy level, e. g. 0 for an `HMesh3d` only consisting of macro elements, 1 if every macro element has children, but no grandchildren, etc.

`level_of_interest`: The maximal hierarchy level that is of interest right now. Recursive traversal of the elements will stop at this level; which is a simple way to reduce computing times.

`boundary_threshold`: The threshold for the error on displaying the boundaries.

Due to their nature, `level_of_interest` and `boundary_threshold` can be adjusted interactively at the graphical user interface.
At last, the `HMESH3D` structure contains an interface to data given *on* the mesh. Here we want to support a situation where different types of data are given at the same time on the same mesh with different value dimensions and other characteristics (example: a piecewise linear, scalar pressure, a piecewise quadratic, vector–valued velocity, and some more), together with some methods to select one of the data values for display. The data characteristics may change between two visualization tasks. Additionally, there may be data characteristics which change from element to element in the same mesh, for example the polynomial degree of the basis functions. To handle all these situations, we use a rather general interface for functions on the mesh. A user supplied procedure evaluates data selected for visualization. Instead of only supporting functions given in terms of a Lagrangian basis, this concept is open for any type of user function such as one of Hermitian type or one only piecewisely defined on several parts of a single element, for example.
Data to be displayed is defined globally on the whole mesh, although the evaluation of data is done on element level (using local coordinates). Thus, the data interface is located at mesh level and described by the following structure:

Discrete Functions

```
typedef struct f_hdata3d
{
  char          *name;
  GENMESH_FDATA  *last, *next;
  int           dimension_of_value;
  int           continuous_data;
  void          *function_data;

  void          (*f)        (ELEMENT3D *, int, double[], double[], void *function_data);
  void          (*f_el_info)(ELEMENT3D *, F_EL_INFO3D *,           void *function_data);

  void          (*get_bounds)(HELEMENT3D *, double *, double *, void *function_data);
  void          (*get_vertex_estimate) (HELEMENT3D *, double *, void *function_data);
  double        (*get_element_estimate)(HELEMENT3D *,           void *function_data);
  double        threshold;
  double        geometry_threshold;
  double        hp_threshold;
  int           hp_maxlevel;

} F_HDATA3D;
```

This is the structure defining a function on an hierarchical mesh. The entries `name` to `f_el_info` are inherited from `Mesh3d`, which is again a subclass of `Gen_Mesh`; due to this inheritance the `F_HDATA3D` has to be downcast to `GENMESH_FDATA3D` in assignments. All entries have to be set by the interface. `threshold`, `geometry_threshold`, `hp_threshold` and `hp_maxlevel` can later be edited on the graphical user interface.
In detail, the entries are:

`name`: a textual description of current data (“pressure”, e.g.), internally used as the name of the function that will appear in selection dialogs on the user interface, `dimension_of_value`: is the dimension of the function values, i. e. 1 for a scalar function, > 1 for a vector valued function,

`continuous_data`: contains the boolean (`TRUE` or `FALSE`) information, if the function is continuous between elements,

`f(el, vindex, coord, val, function_data)`: the data function itself generating `dimension_of_value` – dimensional values at a position which has to be specified in local coordinates on element `el`. This function may also be called with parameters `coord=NULL` and a local index `vindex` of one of the element's vertices.

`f_el_info(el, el_info, function_data)`: fills structure `el_info` with local information about data, such as polynomial degree for use by routines which adaptively choose the display resolution, e.g.

`last, next`: to build a doubly linked list of data descriptions.

`function_data`: optional pointer, passed on every function call, in which the user can store information needed to calculate the function values or to identify the function in the case of dynamical allocation.

`get_bounds(el, min, max, function_data)`: calculates the lower– and upper bound of the function values in an element. If this cannot be determined in an efficient way, return $-\infty$ and $+\infty$.

`get_vertex_estimate (el, vertex_est, function_data)`: calculates the error estimator for all vertices of `el`, filling `vertex_est` with these values.

`get_element_estimate (el, function_data)`: returns the error estimator for the children, thus determining, if the element is to be refined.

`threshold`, `geometry_threshold`, `hp_threshold`: are the thresholds for function, geometry and on–element subdivision in case of higher polynomial degrees, respectively. If an error indicator is above its threshold, refinement will be attempted.

`hp_maxlevel`: is the maximal refinement level for on–element subdivision in case of higher polynomial degrees. Its name resembles *hp–methods*.

The `f_data` entry in the `HMESH3D` structure gives access to data which is currently selected for display. By use of the doubly linked list, an interactive choice between different data is possible.
In the end, the `HMESH3D` structure contains a data pointer `user_data`. This simplifies the simultaneous handling of multiple mesh3d data sets, if for any reason one does not want to create **GRAPE** subclasses of `HMesh3d`.

```
typedef struct f_hel_info3d { int polynomial_degree; } F_HEL_INFO3D;
```

The structure, `f_el_info()` has to fill with information about the function. Currently the only entry is

`polynomial_degree`, which is the function's polynomial degree on the elements, i. e. 1 for a piecewise linear function.

2 Writing Display Methods for the Classes `HMesh2d` and `HMesh3d`

2.1 Introduction

The GRAPE-library contains a large variety of methods for visualizing data given on hierachical grids. On the one hand there are methods for analysing the grid itself on the other hand functions defined on the grid can be presented in different ways. However, not all possible methods and algorithms for extracting and visualizing data can be included in advance. So in accordance with the open and extensible concept of GRAPE the user can write his own display methods. How to do this and which tools exist in GRAPE to facilitate this is the scope of the following sections.
In general any display method will have the following structure:

- Preparation, includes getting the object to be displayed, getting a graphic device, initializing interactive elements with which parameters of the display method can be altered and possibly getting additional tools as for example the colorbar.
- Data extraction, by means of the interface functions of the Hmesh – first-macro, next-macro, etc. – and the access function of the F_hdata-structure the necessary data is extracted. As this process is very similar in many display methods there are powerfull tools for the most common extraction requirements.
- Rendering, finally using the functionality of the graphic device the extracted data are rendered. This part of the display method is almost always specific for it and thus it is the part where except for the routines provided in the graphic device no other tools are available.

In the following each of the steps mentioned above is discussed in detail. Once the user has written a display-method it must be added to the appropriate class as described in [38]. In order to be able to choose the display method in the manager a naming convention must be complied with.
Most examples in this section are based on the class `HMesh2d`. However, the corresponding class `HMesh3d` is similar and most of the examples work also in the 3D-case.

2.2 Preparation

The task of the preparational part of the display method is to provide access to the necessary objects and data structures. As an example the beginning of the hmesh2d isoline display method is printed below. In addition to the `HMesh2d` object the isoline method needs access to the function values. Therefore a pointer to a `F_HDATA2D` structure is initialized. As the isolines are colored according to their isovalue a colorbar is neccessary. Besides the obligatory graphic device the use of a lens is prepared. For more details concerning the different objects see their respective documentation.

```
HMESH2D* hmesh2d_isoline_disp(void)
{
  HMESH2D*       hmesh;
  F_HDATA2D*     f_hdata2d;
  COLORBAR*      colorbar;
  GRAPHICDEVICE* device;

  static HMESH2D_LENS* lens = NULL;

  /* get hmesh and function */

  hmesh = (HMESH2D*)START_METHOD(G_INSTANCE);
  ALERT(hmesh,"hmesh2d-isoline: No hmesh!",END_METHOD(NULL));

  f_hdata2d = (F_HDATA2D*)GRAPE(hmesh,"get-function")
    ("scalar","scalar","vector","default",NULL);
  ALERT(f_hdata2d,"hmesh2d-isoline-select: No function!",
    END_METHOD(NULL));

  /* get colorbar and graphicdevice */

  colorbar = (COLORBAR*)GRAPE(Colorbar,"get-stdcolorbar")
    (hmesh2d_isoline_disp,"isoline-disp");

  device = (GRAPHICDEVICE*)GRAPE(GraphicDevice,"get-stddev")();

  /* get lens */

  if(!lens)
```

```
    lens = (HMESH2D_LENS*)GRAPE(Hmesh2d_Lens,"get-lens")
      ("lens for hmesh2d-isoline");


  ...
}
```

Often the visualiztion algorithms implemented in display methods depend on parameters that allow the user to manipulate the resulting picture. To make this possible interactive elements must be added to the manager. Below an excerpt of the display method hmesh2d-shrink-disp is printed. This methods has one parameter which controls the size of the shrunk elements.

```
HMESH2D* hmesh2d_shrink_disp(void)
{
  MANAGER* mgr;
  ...
  static GROUP*  group           = NULL;
  static SLIDER* shrink_size_ctl = NULL;
  static double  shrink_size     = 0.95;
  ...

  /* get manager */

  mgr = (MANAGER*)GRAPE(Manager,"get-stdmgr")();

  /* get interactives */

  if(!shrink_size_ctl)
    shrink_size_ctl =
       (SLIDER*)new_item(Slider,
                         I_Label,"shrink-factor",
                         I_Var,&shrink_size,dfDouble,
                         I_MinMax,0.0,1.0,
                         I_Scale, 1.0,
                         I_Offset,0.0,
                         I_RSizeX,0.5,I_SizeY,1.0,
                         I_End);
  if(!group)
    group = (GROUP*)new_item(Group,
                             I_Border,bfBorder|bfTitle,
                             I_Name,"hmesh2d-shrink",
                             I_Size,12.0,1.25,
                             I_Item,shrink_size_ctl,
                             I_End);

  if(GRAPE(mgr,"new-handle")(hmesh2d_shrink_disp,1))
    GRAPE(mgr,"add-inter")(group);

  ...
```

2.3 Data Extraction

2.3.1 Overview

In this sections the tools that are helpfull for extracting data are presented. Although the context is to write display methods the tools are also usefull for calculating error estimators, norms of functions and for many other purposes.

There are exceptions, but most display methods extract the necessary data in the following way: The hierachical tree is traversed via the interface functions and on any leaf the data are fetched, calculated and rendered. As hanging nodes may occur in an adaptive traverse, hierachical projection is needed to assure continuity.
To support this, there are functions in GRAPE – namely `g_traverse_hmesh2d()` and `g_traverse_hmesh3d()` – that implement such an traverse. Among other things these routines need three call-back-functions as arguments.

- `projection`; in this routine the necessary data on the currrent element are collected and if appropriate interpolated.
- `test_if_proceed`; this functions controlls the traverse. It is decided whether to proceed on the next finer level or to stop on the current element and in the latter case whether to render on the current element or not. The outcome may be based on the projected data or for example on the error estimators included in the `HMesh`-interface.
- `action_on_element`; this functions should contain the code for rendering the data on an element.

Further arguments of `g_traverse_hmesh2d()` are the `Hmesh2d` object, a pointer to a `HM2_GENERAL` variable, the traverse mode, the level of interest, an array of pointers to the element data, a pointer to other arbitrary data possibly needed in the `action_on_element` routine and a mesh element flag.
The traverse mode, an variable of type integer, determines on which elements the `action_on_element` routine may be called. Possible values are `G_LEAVES`, `G_PREFIX` and `G_POSTFIX`. If the traverse mode is `G_LEAVES` the action is at most called on a leaf of the hierachical tree. In contrast if the traverse mode is `G_PREFIX` or `G_POSTFIX` the `action_on_element` routine may be called on all elements of the tree. In the case of `G_PREFIX` the `action_on_element` routine is called first on the parent element and after that the children are considered. Whereas in the case of `G_POSTFIX` only after the complete subtree of an element is traversed the `action_on_element` routine is called on the element itself.
The level of interest is the maximum depth of the hierachical traverse. In most display methods this should be the level of interest stored in the `Hmesh2d` object. Since this entry can be manipulated by the user via the `uif-hm2`-project it is not neccessary that display methods have an interactive element controlling it.
The interface functions of the `Hmesh2d` allow to choose which entries of the `HELEMENT2D` structure are filled by the interface. In the same way this choice can be made for the whole traverse with the mesh element flag.
Below the exact prototype of the traverse routine is printed. The other arguments are explained in the next section.

```
void g_traverse_hmesh2d(
 HMESH2D*     hmesh,
 HM2_GENERAL* general,
 int          traverse_mode,
 int          level_of_interest,
 int      (*projection)       (HELEMENT2D*  helement,
                               HM2_GENERAL* general,
                               void*        element_data,
                               void*        parent_data),
 int      (*test_if_proceed)  (HELEMENT2D*  helement,
                               HM2_GENERAL* general,
                               void*        element_data),
 int      (*action_on_element)(HELEMENT2D*  helement,
                               HM2_GENERAL* general,
                               void*        element_data,
                               void*        action_arg),
 void**   element_data,
 void*    action_arg,
 MESH_ELEMENT_FLAGS mesh_element_flag)
```

2.3.2 Preparing the traverse

There are two different data structures used by the traverse functions. One structure is defined by the user and contains the element specific data. The other is of the type `HM2_GENERAL` and consists of pointers to objects and data structures that usually are needed in the `action_on_element`-routine but are not element specific, e.g. a pointer to the graphic device or the colorbar. The definition of this structure is given below.

```
 typedef struct hm2_general{
  struct graphicdevice* dev;
  struct colorbar*      colorbar;
  struct hmesh2d_lens*  lens;
  F_HDATA2D*            f_hdata2d;
} HM2_GENERAL;
```

Note that it is not compulsory to fill all entries of this structure. If for example the `action_on_element`-routine does not need the colorbar the entry may be ignored by the user.
The user defined data structure with the element specific data may contain vertex coordinates, nodal function values or color values. The data structure used by the isoline display method for example employs all of the entries:

```
typedef struct hm2_coord_color_fct_data {
  VEC3   vertex_coord[MESH2D_MAX_VERTEX];
  VEC3   vertex_color[MESH2D_MAX_VERTEX];
  double vertex_value[MESH2D_MAX_VERTEX];
  int    in_lens_area;
} HM2_COORD_COLOR_FCT_DATA;
```

Memory for an array of these structure variables must be allocated. The length of this array is given by the maximal depth of the hierachical tree. As an example we have again printed part of the display method `hmesh2d-shrink-disp`.

```
HMESH2D* hmesh2d_shrink_disp(void)
{
  ...
  HM2_GENERAL      general;
  HM2_COORD_DATA** hm2_shrink_data;

  int i,max;

  /* get hmesh, graphic device, ...  and initialize interactives */
  ...
  /* prepare mesh-traverse */

  max = hmesh->level_of_interest + 1;

  general.dev  = dev;
  general.lens = lens;

  G_MEM_ALLOC(hm2_shrink_data,max);

  for(i=0;i<max;i++)
    G_MEM_ALLOC(hm2_shrink_data[i],1);

  /* traverse the mesh */

  g_traverse_hmesh2d(hmesh,
                     &general,
                     G_LEAVES,
                     hmesh->level_of_interest,
                     hm2_coord_projection,
                     hm2_geom_test_if_proceed,
                     hm2_shrink_draw_element,
                     (void**)hm2_shrink_data,
                     NULL,
                     hefVertex|hefVinh);

  /* free memory */

  for(i=0;i<max;i++)
    G_MEM_FREE(hm2_shrink_data[i],1);

  G_MEM_FREE(hm2_shrink_data,max);

  END_METHOD(hmesh);
}
```

The functions `hm2_coord_projection`, `hm2_geom_test_if_proceed` and `hm2_shrink_draw_element` must be defined somewhere else. The GRAPE-library contains several projection and test-if-proceed routines that can be reused and are explained in the next sections.

2.3.3 The projection routine

The purpose of the projection routine is to fill the element specific data in the element data structure. In the example above this was of the type `HM2_COORD_DATA` and contained only the vertex coordinates of the respective element. The data of the current element may depend on the data in the `HELEMENT2D` and the element data of the parent element. Additionally the entries of the `HM2_GENERAL`-structure may play a role in assembling and calculating the data on the current element.
The GRAPE library features several projection routines that can be reused by any other display method. In the 2D-case these are:

- `hm2_coord_simpl_projection`
- `hm2_coord_lens_projection`
- `hm2_scalar_fct_simpl_projection`
- `hm2_scalar_fct_lens_projection`
- `hm2_vec_fct_simpl_projection`
- `hm2_vec_fct_lens_projection`
- `hm2_coord_color_simpl_projection`
- `hm2_coord_color_lens_projection`
- `hm2_coord_color_fct_simpl_projection`
- `hm2_coord_color_fct_lens_projection`

These routines use the following data structure as element data:

```
typedef struct hm2_coord_data {
  VEC3 vertex_coord[MESH2D_MAX_VERTEX];
  int   in_lens_area;
} HM2_COORD_DATA;
```

```
typedef struct hm2_scalar_fct_data {
  double vertex_value[MESH2D_MAX_VERTEX];
  int    in_lens_area;
} HM2_SCALAR_FCT_DATA;

typedef struct hm2_vec_fct_data {
  VEC3  vertex_coord[MESH2D_MAX_VERTEX];
  VEC3  vertex_value[MESH2D_MAX_VERTEX];
  int   in_lens_area;
} HM2_VEC_FCT_DATA;

typedef struct hm2_coord_color_data {
  VEC3  vertex_coord[MESH2D_MAX_VERTEX];
  VEC3  vertex_color[MESH2D_MAX_VERTEX];
  int   in_lens_area;
} HM2_COORD_COLOR_DATA;

typedef struct hm2_coord_color_fct_data {
  VEC3   vertex_coord[MESH2D_MAX_VERTEX];
  VEC3   vertex_color[MESH2D_MAX_VERTEX];
  double vertex_value[MESH2D_MAX_VERTEX];
  int    in_lens_area;
} HM2_COORD_COLOR_FCT_DATA;
```

The names of the projection routines indicate which data are calculated: `_coord_` means that vertex coordinates are compiled, `_vec_fct_` means that vector valued functions are calculated and so on.
Within these projection routines the vertex error estimators are used to decide wether a given datum has to be interpolated from parent data in order to assure continuity or if it has to be filled with the original value provided by the `HMESH` interface.

2.3.4 The `test-if-proceed` routine

As mentioned above this functions controlls the traverse. It may return each of the three values `G_HM_NOACTION`, `G_HM_PROCEED` or `G_HM_ACTION`. If `G_HM_ACTION` is returned the traverse is stopped on the current element and the `action_on_element` routine is called. Whereas if `G_HM_NOACTION` is returned the traverse is stopped on the current element without calling the `action_on_element` routine. In contrast `G_HM_PROCEED` causes the traverse to go on to the next finer level. It is important to understand that the traverse mode determines on which elements the `action_on_element` routine may be called (on all nodes of the tree or only on the leaves) but the result of `test_if_proceed` determines when the `action_on_element` routine is actually called.
The arguments of the `test_if_proceed` routine are the `HELEMENT2D`, the `HM2_GENERAL`-structure and the element data of the current element. For an example see the `hm2_fct_lens_test_if_proceed` routine at the end of this section.
Typically the `test_if_proceed` routine compares an element error indicator to an appropriate threshold. In the 3D-case other intersection tests are also a common task performed in this routine. In the GRAPE-library several standard routines of this kind are incorporated. In the 2D-case these are:

- `hm2_geom_simpl_test_if_proceed`
- `hm2_geom_lens_test_if_proceed`
- `hm2_fct_simpl_test_if_proceed`
- `hm2_fct_lens_test_if_proceed`
- `hm2_geom_fct_simpl_test_if_proceed`

- hm2_geom_fct_lens_test_if_proceed

As the names indicate either the geometry error indicators, the function error indicators or both are compared to their respective threshold. Additionally the functionality of the lens is supported or not.

```
int hm2_fct_lens_test_if_proceed(HELEMENT2D* helement,
                                 HM2_GENERAL* general,
                                 void* el_data)
{
  HMESH2D*      hmesh     = (HMESH2D*)helement->mesh;
  F_HDATA2D*    f_hdata2d = general->f_hdata2d;
  HMESH2D_LENS* lens      = general->lens;

  double   error,lens_error;
  double   lens_threshold;

  /* get element error estimator */

  if(f_hdata2d->get_element_estimate)
    error = f_hdata2d->get_element_estimate(helement,
                                            f_hdata2d->function_data);
  else
    error = f_hdata2d->threshold + 1.0;

  /* lens condition */

  lens_threshold = lens->get_lens_element_threshold(helement,lens);

  if(hmesh->get_lens_element_estimate)
    lens_error = hmesh->get_lens_element_estimate(helement,
                                                  lens->lens_data);
  else
    lens_error = lens_threshold + 1.0;

  /* compare with threshold */

  if(error < f_hdata2d->threshold && lens_error < lens_threshold)
    return G_HM_ACTION;
  else
    return G_HM_PROCEED;
}
```

2.3.5 The action-on-element routine

This routine performs the actual rendering and displaying on a given element. The arguments are the HELEMENT2D, the HM2_GENERAL-structure, the element data of the current element and a pointer to arbitrary, user defined data. The last argument is of the type void* and can be used to pass data from the display method to the action_on_element routine.
Below the hm2_disp_draw_element routine is printed as an example.

```
static int hm2_disp_draw_element(HELEMENT2D*  helement,
                                 HM2_GENERAL* general,
                                 void*        el_data,
                                 void*        action_arg)
{
  GRAPHICDEVICE*          dev    = general->dev;
  HMESH2D*                hmesh  = (HMESH2D*)helement->mesh;
  HELEMENT2D_DESCRIPTION* descr  =
      (HELEMENT2D_DESCRIPTION*)helement->descr;
  HM2_COORD_DATA* element_data = (HM2_COORD_DATA*)el_data;

  VEC3* vertex_coord = element_data->vertex_coord;
  int   i,number_of_vertices = descr->number_of_vertices;


  if(dev->grid_patch == G_GRID){
    dev->move(vertex_coord[number_of_vertices-1]);
    for(i=0;i<number_of_vertices;i++)
      dev->draw(vertex_coord[i]);
  }
  else{
    VEC3 normal;

    g_vec3_get_normal_to_plane_quietly(normal,
                                       vertex_coord[0],
                                       vertex_coord[1],
                                       vertex_coord[2]);
    dev->begin_patch();
    dev->patch_normal(normal);

    for(i=0;i<number_of_vertices;i++)
      dev->patch_vertex(vertex_coord[i]);

    dev->end_patch();
  }

  return TRUE;
}
```

3 Hierarchical Searching using the Interface

Particle tracing methods which generate and visualize particle lines, stream surfaces ore moving clouds of particles are important tools for the examination of velocity fields. The efficiency of these methods depends senitively on searching algorithms. Let us suppose that h and $\Delta t|u_{\max}|$ are of the same size in the corresponding integration method. Fig 2 sketches a typical particle line on a 2D grid. The basic task is to locate points on the grid, which are computed by the corresponding ODE solver. For each new position we need the grid element and the corresponding local coordinates to evaluate the numerical velocity at that position. It is obvious, that a local algorithm should be used in this searching process. This local method can be provided with an inital guess, which typically is the result of the last timestep. Figure 3 illustrates two alternative schemes. On the one hand, we can proceed recursively up and down. Thereby we first move from the guess positions fine grid element successively to coarser elements until the destination position is in the current element or we have reached the macro grid level. Then we zoom into finer grid level until the finest element which contains the destination position has been reached. On a other hand, we can start on the macro level, search for a coarse element containing the destination position and then perform the above zooming operation. Especially on hierarchical grids of a larger depth the first method is obviously the better choice. I. e. in average about four level changes (two up and two down respectively) are necessary for a 2D grid consisting of rectangles to locate the new point of the particle line under the above assumption on the step sizes. The kernel of the second algorithm could be implemented in the following way:

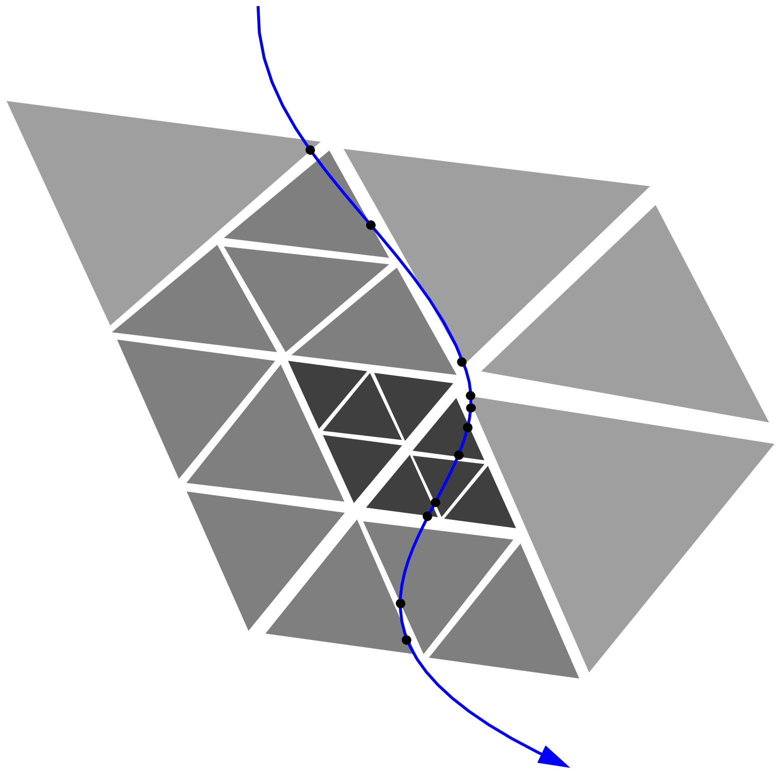


Figure 2: Typical particle line

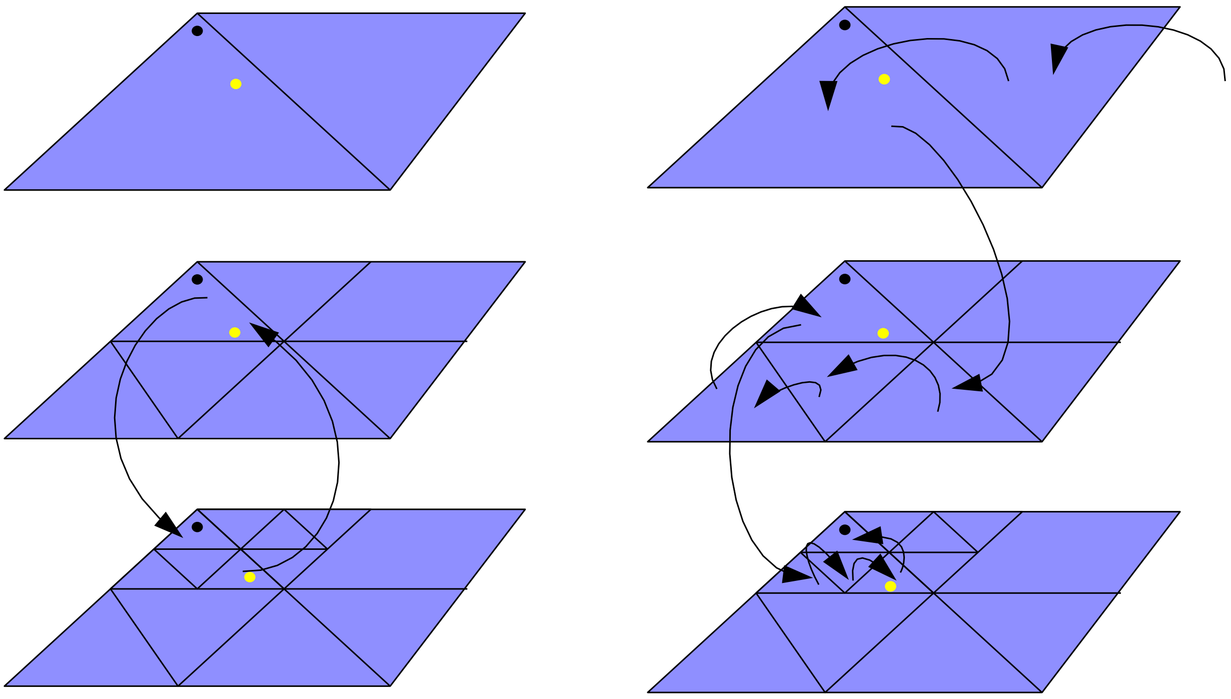


Figure 3: Different searching strategies

```
element = self->first_macro(hefAll);
element->descr->world_to_coord((ELEMENT*)element,point,local_coord);
while(element->descr->check_inside((ELEMENT*)element,local_coord)==false){
 element = self->next_macro(element,hefAll);
 }

while((help_element = element->first_child(element,hefAll))!=NULL){
 element = help_element;

 element->descr->world_to_coord((ELEMENT*)element,point,local_coord);
 while(element->descr->check_inside((ELEMENT*)element,point)==false){
   element = element->next_child(element,hefAll);
  }
 }
```

To improve efficiency the new interface routines

```
void coord_of_parent(HELEMENT3D *element, double *local_coord,
        double *local_coord_of_parent);
```

and

```
HELEMENT3D *select_child(HELEMENT3D *element, double *local_coord,
        double *local_coord_of_child,HELEMENT3D_FLAGS hefall)
```

have been added. *coord_of_parent()* transforms the local coordinates of a point in an element to that one of the parent element. *select_child()* returns the child element containing a point already known to be inside the parent element and calculates its local coordinates corresponding to the child element. This allows very fast and direct access to elements of arbitrary nested hierarchical grids. With these functions at hand, the kernel for the first algorithm look as follows:

```
if(element != NULL){
 element->descr->world_to_coord((ELEMENT*)element,point,local_coord);
 while((-1 != element->desc->check_inside((ELEMENT3D *)element,local_coord)){
        &&(e[0]->parent!=NULL))
  help_element = element;
  ((HELEMENT3D_DESCRIPTION*)(element->descr))->coord_of_parent
(element, local_coord,local_coord);
  element = element->parent;
  hmesh->free_element((ELEMENT3D*)help_element);
  }
 while((help_element = hmesh->select_child
(element,local_coord,local_coord,hefAll))!=NULL)
  {element = help_element;}
 }
```

This algorithm is implemented in the method

```
HMESH3D *hmesh3d_search(VEC3 point, double *local_coord, HELEMENT3D **element)
```

which uses **element* as the element of the intial guess, to locate a particle at position *point* on the grid hierarchy.

4 An Interface Recipe

4.1 Introduction

In the following we will demonstrate which steps the user has to pass to get from a general interface mask to a complete hierarchical interface. In each step we will first present the code from the interface mask (marked by horizontal lines) and afterwards discuss the neccessary modifications. The recipe and the example cover the 2D-case. The 3D-case is analogous.

The interface consists of the following routines:

- HMESH2D-routines
  - get_new_hmesh()
  - Routines to traverse the hierarchical mesh
    - interface_first_macro()
    - interface_next_macro()
    - interface_first_child()
    - interface_next_child()
    - interface_select_child()
    - interface_first_element()
    - interface_next_element()
  - Handling interface_elements
    - get_interface_element()
    - free_interface_element()
    - fill_interface_element_with_user_macro_element()
    - fill_interface_element_with_user_child_element()
  - Helement Description
    - element_world_to_coord()
    - element_coord_to_world()
    - helement_description_check_inside()
    - helement_description_neighbour()
    - helement_description_boundary()
    - helement_description_coord_of_parent()
  - Geometry error estimator
    - get_geometry_vertex_estimate()
    - get_geometry_element_estimate()
- Routines for functions on hierachical meshes
  - get_new_fhdata()
  - data_access_function()
  - local_information()
  - get_bounds()
  - Function error estimator
    - get_function_vertex_estimate()
    - get_function_element_estimate()

In order to illustrate the single steps we introduce a sample data structure. As the interface uses its own data structures we have to distinguish three kinds of data structures:

- the data structure of the user
- the internal GRAPE data structures
- the interface data structures

In the example the user data structures are called BISMESH, BELEMENTs and BELEMENT_MACRO. The task of the interface is to translate these structures into the respective internal GRAPE structures HMESH2D and HELEMENT2D. The interface employs so called INTERFACE_ELEMENTs to generate the HELEMENT2Ds.

4.2 The Bismesh Structure

Our sample data structure will describe a triangulation of a given domain that is refined by bisection starting from a macro triangulation. Each element is represented in the following data structure called BELEMENT:

```
typedef struct belement{
  struct belement* neighbour[3];
  struct belement* child[2];
  struct belement* parent;

  int newnode;
  int flag;
} BELEMENT;
```

Here newnode contains the global index of the node that is created while the element is being bisected. The flag is used to decide which element has to be refined. Elements of the macro triangulation called BELEMENT_MACRO are basically the same as the BELEMENTs. They only have some additional information included:

```
typedef struct belement_macro{
  BELEMENT* neighbour[3];
  BELEMENT* child[2];
  BELEMENT* parent;

  int newnode;
  int flag;

  int node[3];

  struct belement_macro* next;
} BELEMENT_MACRO;
```

The BELEMENT_MACROs are chain linked via the next pointer while the node array contains the global indices of the vertices.

Global information regarding the bisection mesh is grouped in the structure BISMESH:

```
typedef struct bismesh{
  int number_macro_points;
  int number_macro_elements;
  int number_points;
  int number_elements;
  int max_level;

  BELEMENT_MACRO* first;

  VEC3* macro_coord;

  double* geo_estimates;
} BISMESH;
```

The pointer first enables us to access the chain of the macro elements while the coordinates of the macro nodes are stored in the array macro_coord. The error estimators concerning the geometry are stored in the array geo_estimates.

4.3 Data Structures of the Interface

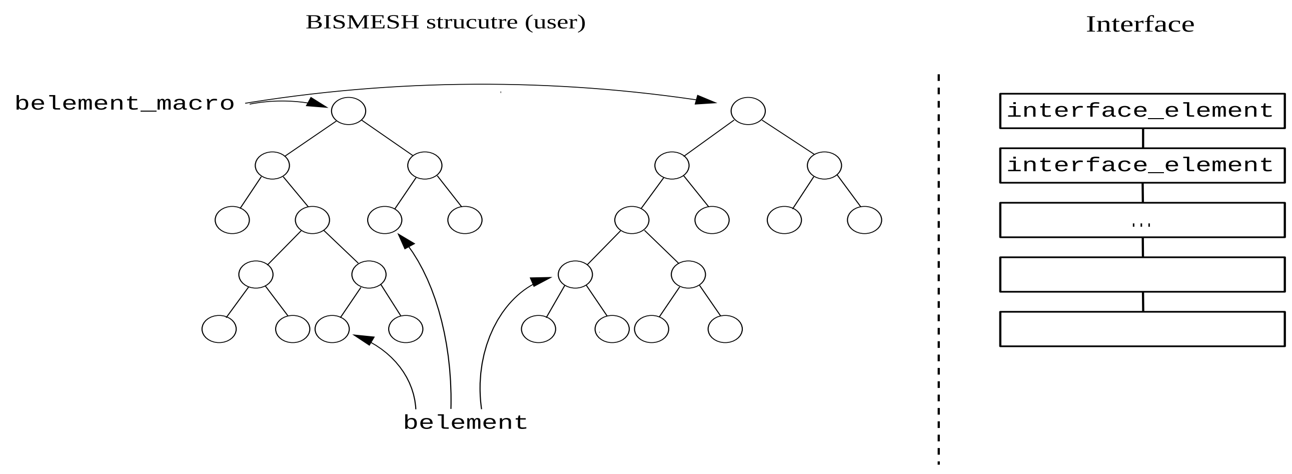
The purpose of a hmesh interface is to translate data stored in a given user format – like the above bismesh structures – into the HMESH2D/HELEMENT2D structures. According to the interface mask the INTERFACE_ELEMENT consists of all data of the HELEMENT2D. These are grouped in HMESH2D_HELEMENT2D_STRUCT. The additional variables support the recursive mesh traversal and the generation of HELEMENT2Ds. Furthermore there may be some entries which depend on the user's data format.

```
typedef struct interface_element {
  HMESH2D_HELEMENT2D_STRUCT;

  USER_ELEMENT* user_element;
  double*       point_coords[MAX_NUMBER_OF_VERTICES_PER_ELEMENT];
  int           level;

  struct interface_element* next;

  ...                              /* additional entries by the user */

} INTERFACE_ELEMENT;
```

In our case the INTERFACE_ELEMENT structure of the mask has the concrete outline:

```
typedef struct interface_element {
  HMESH2D_HELEMENT2D_STRUCT;

  BELEMENT*     belement;
  double*       point_coords[MAX_NUMBER_OF_VERTICES_PER_ELEMENT];
  int           level;

  struct interface_element* next;

  VEC3          coord_of_new_node;

} INTERFACE_ELEMENT;
```

The INTERFACE_ELEMENTs are organized in a free list which is linked via the next pointer. The coordinates of the point that is created while the element is being bisected are stored in coord_of_new_node. Although the routines for traversing the mesh have pointers to HELEMENT2Ds as arguments and return values, in the inteface these pointers will be casted pointers to INTERFACE_ELEMENTs. In an object oriented setting one would apply the subclass concept here.



4.4 get_new_hmesh()

The initialization routine of the interface mask gets a new instance of `HMESH2D` and fills in the neccessary data. In the interface mask it looks like that:

```
HMESH2D* get_new_hmesh(USER_MESH* user_mesh,...)
{
  HMESH2D* hmesh;

  hmesh = (HMESH2D*)GRAPE(HMesh2d,"new-instance")("name of hmesh");
  ASSURE(hmesh,"get_new_hmesh: can't get new hmesh instance",return NULL);

  hmesh->first_macro          = interface_first_macro;
  hmesh->next_macro           = interface_next_macro;
  hmesh->first_child          = interface_first_child;
  hmesh->next_child           = interface_next_child;
  hmesh->select_child         = interface_select_child;
  hmesh->first_element        = interface_first_element;
  hmesh->next_element         = interface_next_element;

  hmesh->max_level              = ...;
  hmesh->max_vindex             = ...;
  hmesh->level_of_interest      = ...;
  hmesh->dimension_of_world     = ...;
  hmesh->max_dimension_of_coord = ...;
  hmesh->max_number_of_vertices = MAX_NUMBER_OF_VERTICES_PER_ELEMENT;
  hmesh->geometry_data          = (void*)...;  /* 2D-case only */


  /* in 2D-case */

  hmesh->get_geometry_vertex_estimate  =
    interface_get_geometry_vertex_estimate;

  hmesh->get_geometry_element_estimate =
    interface_get_geometry_element_estimate;


  /* fill the helement description */

  helement_description->number_of_vertices = ...;
  helement_description->dimension_of_coord = ...;
  helement_description->coord              = ...;
  helement_description->parametric_degree  = ...;
  helement_description->world_to_coord     = ...;
  helement_description->coord_to_world     = ...;

  helement_description->check_inside    = helement_description_check_inside;
  helement_description->neighbour       = helement_description_neighbour;
  helement_description->boundary        = helement_description_boundary;
  helement_description->coord_of_parent = helement_description_coord_of_parent;

  /* inheritance rules */

  inheritance_rule[0]->np      = ...; /* number of parent points */
  inheritance_rule[0]->pindex  = ...;
  inheritance_rule[0]->pweight = ...;

  ...

  inheritance_rule[n]->np      = ...; /* number of parent points */
  inheritance_rule[n]->pindex  = ...;
  inheritance_rule[n]->pweight = ...;

  return hmesh;
}
```

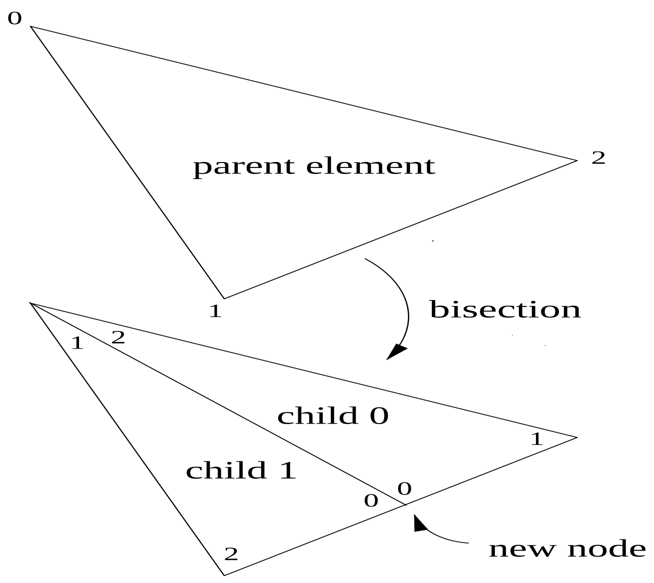
In our example this becomes:

```
HMESH2D* get_new_hmesh(BISMESH* bismesh)
{
  HMESH2D* hmesh;

  hmesh = (HMESH2D*)GRAPE(HMesh2d,"new-instance")("my sample bismesh");
  ASSURE(hmesh,"get_new_hmesh: can't get new hmesh instance",return NULL);
  hmesh->first_macro          = interface_first_macro;
  hmesh->next_macro           = interface_next_macro;
  hmesh->first_child          = interface_first_child;
  hmesh->next_child           = interface_next_child;
  hmesh->select_child         = interface_select_child;
  hmesh->first_element        = interface_first_element;
  hmesh->next_element         = interface_next_element;

  hmesh->max_level              = bismesh->max_level;
  hmesh->max_vindex             = bismesh->number_points;
  hmesh->user_data              = (void*)bismesh;
  hmesh->level_of_interest      = 1; /* can be changed interactively later */
  hmesh->dimension_of_world     = 3;
  hmesh->max_dimension_of_coord = 3;
  hmesh->max_number_of_vertices = MAX_NUMBER_OF_VERTICES_PER_ELEMENT;
  hmesh->geometry_data          = NULL;


  /* in 2D-case, see I.5 */

  hmesh->get_geometry_vertex_estimate  =
    get_geometry_vertex_estimate;

  hmesh->get_geometry_element_estimate =
    get_geometry_element_estimate;


  /* fill the helement description */

  helement_description.number_of_vertices = 3;
  helement_description.dimension_of_coord = 3; /* number of local coords
                                                  on an element */
  helement_description.coord              = local_coordinate_system;
  helement_description.parametric_degree  = 1;
  helement_description.world_to_coord     = triangle_world_to_coord;
  helement_description.coord_to_world     = triangle_coord_to_world;
  helement_description.check_inside       = triangle_check_inside;

  helement_description.neighbour       = helement_description_neighbour;
  helement_description.boundary        = helement_description_boundary;
  helement_description.coord_of_parent = helement_description_coord_of_parent;

  /* inheritance rules */

  inheritance_rule_in_child_0[0] = vinherit_point_0;
  inheritance_rule_in_child_0[1] = vinherit_point_1_in_child_0;
  inheritance_rule_in_child_0[2] = vinherit_point_2_in_child_0;

  inheritance_rule_in_child_1[0] = vinherit_point_0;
  inheritance_rule_in_child_1[1] = vinherit_point_1_in_child_1;
  inheritance_rule_in_child_1[2] = vinherit_point_2_in_child_1;

  return hmesh;
}
```

The inheritance rules defined at the bottom describe how points in a child element can be expressed as a convex combination of points in the parent element.

For the bisection of triangles there are the following six inheritance rules:

```
static VINHERIT inheritance_rule_in_child_0[3];
static VINHERIT inheritance_rule_in_child_1[3];

static int        pindex_point_0[2] = {1  ,2  };
static double    pweight_point_0[2] = {0.5,0.5};
static VINHERIT vinherit_point_0    = {2,pindex_point_0,pweight_point_0};

static double pweight_point_1_or_2[1] = {1.0};

static int        pindex_point_1_in_child_0[1] = {2};
static VINHERIT vinherit_point_1_in_child_0    =
        {1,pindex_point_1_in_child_0,pweight_point_1_or_2};

static int        pindex_point_2_in_child_0[1] = {0};
static VINHERIT vinherit_point_2_in_child_0    =
        {1,pindex_point_2_in_child_0,pweight_point_1_or_2};

static int        pindex_point_1_in_child_1[1] = {0};
static VINHERIT vinherit_point_1_in_child_1    =
        {1,pindex_point_1_in_child_1,pweight_point_1_or_2};
```



```
static int        pindex_point_2_in_child_1[1] = {1};
static VINHERIT vinherit_point_2_in_child_1    =
        {1,pindex_point_2_in_child_1,pweight_point_1_or_2};
```

The meaning of the `VINHERIT` are discussed in detail in 1.

4.5 Routines to Traverse the hmesh

The following routines are used by GRAPE to traverse the mesh. The main modification from the code in the interface mask is to provide the respective element of the user data structure, i.e. in our case the `BELEMENT` or `BELEMENT_MACRO`.

interface_first_macro()

This routine gets the first macro element of the user mesh. In the interface mask it looks like that:

```
static HELEMENT2D* interface_first_macro(HMESH2D*           hmesh,
                                         MESH_ELEMENT_FLAGS flags)
{
  USER_MACRO_ELEMENT* user_macro        = ...;
  INTERFACE_ELEMENT*  interface_element = get_interface_element(hmesh);

  fill_interface_element_with_user_macro_element(interface_element,
                                                 user_macro,
                                                 flags);

  return (HELEMENT2D*)interface_element;
}
```

In our bismesh structure we have a pointer to the first macro element at hand. We use the interface routines `get_interface_element()` to request an empty `INTERFACE_ELEMENT` from the free list and `fill_interface_element_with_user_macro_element()` to write the data of `belement_macro` in the `INTERFACE_ELEMENT`.

```
static HELEMENT2D* interface_first_macro(HMESH2D*           hmesh,
                                         MESH_ELEMENT_FLAGS flags)
{
  BISMESH*           bismesh           = (BISMESH*)(hmesh->user_data);
  BELEMENT_MACRO*    belement_macro    = bismesh->first;
  INTERFACE_ELEMENT* interface_element = get_interface_element(hmesh);

  fill_interface_element_with_user_macro_element(interface_element,
                                                 belement_macro,
                                                 flags);

  return (HELEMENT2D*)interface_element;
}
```

interface_next_macro()

This routine gets the next macro element of a given macro element. If there is no next macro element the used `INTERFACE_ELEMENT` is returned to the free list.

```
static HELEMENT2D* interface_next_macro(HELEMENT2D*        helement,
                                        MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT*  interface_element = (INTERFACE_ELEMENT*)helement;
  USER_MACRO_ELEMENT* user_macro        = ...;

  if(user_macro){
    fill_interface_element_with_user_macro_element(interface_element,
                                                   user_macro,
                                                   flags);
  }
  else{
    free_interface_element(interface_element);
    interface_element = NULL;
  }

  return (HELEMENT2D*)interface_element;
}
```

This translates into:

```
static HELEMENT2D* interface_next_macro(HELEMENT2D*        helement,
                                        MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_element = (INTERFACE_ELEMENT*)helement;
  BELEMENT_MACRO*    belement_macro    =
    ((BELEMENT_MACRO*)interface_element->belement)->next;

  if(belement_macro){
    fill_interface_element_with_user_macro_element(interface_element,
                                                   belement_macro,
                                                   flags);
  }
  else{
    free_interface_element(interface_element);
    interface_element = NULL;
  }

  return (HELEMENT2D*)interface_element;
}
```

interface_first_child()

This routine gets the first child element of a given element.

```
static HELEMENT2D* interface_first_child(HELEMENT2D*        helement,
                                         MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_child_element;
  INTERFACE_ELEMENT* interface_parent_element = (INTERFACE_ELEMENT*)helement;

  USER_ELEMENT* user_element = ...;
  int level_of_interest = ((HMESH2D*)helement->mesh)->level_of_interest;
  if(user_element && (interface_parent_element->level < level_of_interest)){

    interface_child_element = get_interface_element((HMESH2D*)helement->mesh);
    fill_interface_element_with_user_child_element(interface_child_element,
                                                   interface_parent_element,
                                                   user_element,
                                                   flags);
    return (HELEMENT2D*)interface_child_element;
  }
  else
    return NULL;
}
```

In our example this reads as follows:

```
static HELEMENT2D* interface_first_child(HELEMENT2D*        helement,
                                         MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_child_element;
  INTERFACE_ELEMENT* interface_parent_element = (INTERFACE_ELEMENT*)helement;

  BELEMENT* belement = interface_parent_element->belement->child[0];
  int level_of_interest = ((HMESH2D*)helement->mesh)->level_of_interest;

  if(belement && (interface_parent_element->level < level_of_interest)){

    interface_child_element = get_interface_element((HMESH2D*)helement->mesh);
    fill_interface_element_with_user_child_element(interface_child_element,
                                                   interface_parent_element,
                                                   belement,
                                                   flags);
    return (HELEMENT2D*)interface_child_element;
  }
  else
    return NULL;
}
```

interface_next_child()

This routine returns the next child element of the parent of the given (child) element. If there is no next child element the used `INTERFACE_ELEMENT` is returned to the free list.

```
static HELEMENT2D* interface_next_child(HELEMENT2D*        helement,
                                        MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_child_element  = (INTERFACE_ELEMENT*)helement;
  INTERFACE_ELEMENT* interface_parent_element =
    (INTERFACE_ELEMENT*)helement->parent;

  USER_ELEMENT* user_element = ...;

  if(user_element){
    fill_interface_element_with_user_child_element(interface_child_element,
                                                   interface_parent_element,
                                                   user_element,
                                                   flags);
    return (HELEMENT2D*)interface_child_element;
  }
  else{ /* there exists no further child\_element */
    free_interface_element(interface_child_element);
    return NULL;
  }
}
```

To adapt this to our BISMESH case we have to modify it as follows:

```
static HELEMENT2D* interface_next_child(HELEMENT2D*        helement,
                                        MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_child_element  = (INTERFACE_ELEMENT*)helement;
  INTERFACE_ELEMENT* interface_parent_element =
    (INTERFACE_ELEMENT*)helement->parent;
  if(interface_child_element->belement ==
     interface_parent_element->belement->child[0]){

    fill_interface_element_with_user_child_element(interface_child_element,
                             interface_parent_element,
                             interface_parent_element->belement->child[1],
                             flags);

    return (HELEMENT2D*)interface_child_element;
  }
  else{
    free_interface_element(interface_child_element);
    return NULL;
  }
}
```

interface_select_child()

The purpose of this routine is explained in 1. In our sample case we have:

```
static HELEMENT2D* interface_select_child(HELEMENT2D* parent_helement,
                                          double*     local_parent_coord,
                                          double*     local_child_coord,
                                          MESH_ELEMENT_FLAGS flags)
{
  HELEMENT2D*        child_helement;
  INTERFACE_ELEMENT* interface_child_element;
  BELEMENT*          belement;

  if(local_parent_coord[1] < local_parent_coord[2]){
    belement = ((INTERFACE_ELEMENT*)parent_helement)->belement->child[0];
    if(belement){
      local_child_coord[0] = local_parent_coord[1] * 2.0;
      local_child_coord[2] = local_parent_coord[0];
      local_child_coord[1] = 1.0 - local_child_coord[0] -local_child_coord[2];
    }
  }
  else{
    belement = ((INTERFACE_ELEMENT*)parent_helement)->belement->child[1];
    if(belement){
      local_child_coord[0] = local_parent_coord[2] * 2.0;
      local_child_coord[1] = local_parent_coord[0];
      local_child_coord[2] = 1.0 - local_child_coord[0] -local_child_coord[1];
    }
  }

  if(belement){
    interface_child_element =
      get_interface_element((HMESH2D*)parent_helement->mesh);

    fill_interface_element_with_user_child_element(interface_child_element,
                             (INTERFACE_ELEMENT*)parent_helement,
                             belement,
                             flags);

    return (HELEMENT2D*)interface_child_element;
  }
  else{
    return NULL;
  }
}
```

interface_first_element()

This is a standard-`mesh2d`-routine and returns the first element on the deepest level described by `hmesh->level_of_interest`. It is independent of the user's data structures and does not need to be changed in any way.

```
static ELEMENT2D* interface_first_element(MESH2D* mesh,
                                          MESH_ELEMENT_FLAGS flags)
{
  HMESH2D*    hmesh    = (HMESH2D*)mesh;
  HELEMENT2D* helement = hmesh->first_macro(hmesh,flags);
  HELEMENT2D* auxiliary_helement;

  INTERFACE_ELEMENT* interface_element = (INTERFACE_ELEMENT*)helement;

  while(auxiliary_helement = hmesh->first_child(helement,flags)){
    helement          = auxiliary_helement;
    interface_element = (INTERFACE_ELEMENT*)helement;
  }

  return (ELEMENT2D*)helement;
}
```

interface_next_element()

Like `interface_first_element()` this is a standard-`mesh2d`-routine and it returns the next element on the deepest level described by `hmesh->level_of_interest`. It is independent of the user's data structures, too. Therefore it does not need to be changed either.

```
static ELEMENT2D* interface_next_element(ELEMENT2D*         element,
                                         MESH_ELEMENT_FLAGS flags)
{
  HMESH2D*    hmesh     = (HMESH2D*)helement->mesh;
  HELEMENT2D* helement  = (HELEMENT2D*)element;
  HELEMENT2D* helement_next;

  if(helement->parent){
      helement_next   = hmesh->next_child(helement,flags);
    if(!helement_next)
      helement_next = (HELEMENT2D*)interface_next_element((ELEMENT2D*)
                                            helement->parent,flags);
  }
  else
    helement_next = hmesh->next_macro(helement,flags);

  if(helement_next){
    while(helement = hmesh->first_child(helement_next,flags))
      helement_next = helement;
  }

  return (ELEMENT2D*)helement_next;
}
```

4.6 Handling INTERFACE_ELEMENTs

Now we list and describe fundamental procedures which support the handling of elements in the interface.

get_interface_element()

This routine returns a new `INTERFACE_ELEMENT`. Either there is an empty one in the free list or a completly new one is generated. As the additional data in the `INTERFACE_ELEMENT` in our example does not need any special memory treatment there are no modifications necessary.

```
static INTERFACE_ELEMENT* get_interface_element(HMESH2D* hmesh)
{
  INTERFACE_ELEMENT* interface_element;

  if(interface_free_list_first_entry){
    /*
       We have an interface_element in our free list. It was generated
       somewhen in the past and freed afterwards. So we can use the
       allocated memory again.
    */

    interface_element               = interface_free_list_first_entry;
    interface_free_list_first_entry = interface_element->next;
  }
  else {
    /*
       There is no interface_element in our free list, so we have to
       generate a new one.
    */

    interface_element =
      (INTERFACE_ELEMENT*)mem_alloc(sizeof(INTERFACE_ELEMENT));

    interface_element->next   = NULL;

    interface_element->mesh   = (MESH2D*)hmesh;
    interface_element->vertex = (double**)interface_element->point_coords;
    interface_element->vindex = int_alloc(MAX_NUMBER_OF_VERTICES_PER_ELEMENT);
    interface_element->vinh   = mem_alloc(MAX_NUMBER_OF_VERTICES_PER_ELEMENT *
                                          sizeof(VINHERIT));
  }

  return interface_element;
}
```

free_interface_element()

If data stored in `interface_element` are no longer used, the `INTERFACE_ELEMENT` can be appended to the free list. The routine does not need to be changed.

```
static void free_interface_element(INTERFACE_ELEMENT* interface_element)
{
  interface_element->next         = interface_free_list_first_entry;
  interface_free_list_first_entry = interface_element;

  return;
}
```

Let us emphasize that all free list entries contain useful initialized values which will be reused in future requests.

fill_interface_element_with_user_macro_element()

This routine fills an `INTERFACE_ELEMENT` with the data of a user macro element.

```
static void fill_interface_element_with_user_macro_element(
                                 INTERFACE_ELEMENT*  interface_element,
                                 USER_MACRO_ELEMENT* user_macro,
                                 MESH_ELEMENT_FLAGS  flags)
{
  int number_of_vertices = ...;
  int i;

  /* element2d-entries */

  if(flags & hefVindex)
    for(i=0;i<number_of_vertices;i++)
      interface_element->vindex[i] = ...;

  interface_element->eindex    = ...;
  interface_element->descr     = (ELEMENT2D_DESCRIPTION*)...;
  interface_element->present   = flags;
  interface_element->user_data = NULL;

  /* helement2d-entries */
interface_element->parent   = NULL;
interface_element->ref_rule = ...;

   /* interface_element-entries */

if(flags & hefVertex)
  for(i=0;i<number_of_vertices;i++)
    interface_element->point_coords[i] = ...;

interface_element->user_element = (USER_MACRO_ELEMENT*)user_macro;
interface_element->level = 0;

return;
}
```

The BISMESH version of it looks like:

```
static void fill_interface_element_with_user_macro_element(
                                 INTERFACE_ELEMENT* interface_element,
                                 BELEMENT_MACRO*    belement_macro,
                                 MESH_ELEMENT_FLAGS flags)
{
  HMESH2D* hmesh         = (HMESH2D*)interface_element->mesh;
  BISMESH* bismesh       = (BISMESH*)hmesh->user_data;
  double norm            = 0.0;
  int number_of_vertices = 3;
  int i;

  /* element2d-entries */

  for(i=0;i<number_of_vertices;i++)
    interface_element->vindex[i] = belement_macro->node[i];

  interface_element->descr     = (ELEMENT2D_DESCRIPTION*)&helement_description;
  interface_element->present   = flags;
  interface_element->user_data = NULL;

  /* helement2d-entries */

  interface_element->parent   = NULL;


  /* interface_element-entries */

  for(i=0;i<number_of_vertices;i++)
    interface_element->point_coords[i] =
      bismesh->macro_coord[belement_macro->node[i]];

  interface_element->belement = (BELEMENT*)belement_macro;
  interface_element->level = 0;

  for(i=0;i<3;i++){
    interface_element->coord_of_new_node[i] =
      0.5 * interface_element->point_coords[1][i] +
      0.5 * interface_element->point_coords[2][i];

    norm += interface_element->coord_of_new_node[i] *
            interface_element->coord_of_new_node[i];
  }

  norm = g_sqrt(norm) / 0.5;

  for(i=0;i<3;i++)
    interface_element->coord_of_new_node[i] /= norm;

  return;
}
```

At the bottom of the routine the coordinates of the new node are calculated. Here we have to take the geometry of the mesh into account. In our example the mesh geometry is simply a sphere with the radius 0.5.

fill_interface_element_with_user_child_element()

This routine serves the purpose of filling an `INTERFACE_ELEMENT` with a non macro element. The necessary data either comes from the respective user element or is derived from the parent `INTERFACE_ELEMENT`.

```
static void fill_interface_element_with_user_child_element(
                                INTERFACE_ELEMENT* interface_child_element,
                                INTERFACE_ELEMENT* interface_parent_element,
                                USER_ELEMENT*      user_element,
                                MESH_ELEMENT_FLAGS flags)
{
  int number_of_vertices = ...;
  int i;

  /* element2d-entries */

  if(flags & hefVindex)
    for(i=0;i<number_of_vertices;i++)
      interface_element->vindex[i] = ...;

  interface_element->eindex    = ...;
  interface_element->descr     = (ELEMENT2D_DESCRIPTION*)...;
  interface_element->present   = flags;
  interface_element->user_data = NULL;

  /* helement2d-entries */

  interface_element->parent = (HELEMENT2D*)interface_parent_element;
  if(flags & hefVinh)
    for(i=0;i<number_of_vertices;i++)
      interface_element->vinh[i] = ...;

  interface_element->ref_rule = ...;

  /* interface_element-entries */

  if(flags & hefVertex)
    for(i=0;i<number_of_vertices;i++)
      interface_element->point_coords[i] = ...;

  interface_element->user_element = user_element;
  interface_element->level = ...;

  return;
}
```

The necessary modifications in our example are listed below:

```
static void fill_interface_element_with_user_child_element(
                                INTERFACE_ELEMENT* interface_child_element,
                                INTERFACE_ELEMENT* interface_parent_element,
                                BELEMENT*          belement,
                                MESH_ELEMENT_FLAGS flags)
{
  double norm = 0.0;
  int number_of_vertices = 3;
  int i;

  /* element2d-entries */

  interface_child_element->vindex[0] =
    interface_parent_element->belement->newnode;

  if(belement == interface_parent_element->belement->child[0]){
    interface_child_element->vindex[1] =
      interface_parent_element->vindex[2];
    interface_child_element->vindex[2] =
      interface_parent_element->vindex[0];
  }
  else{
    interface_child_element->vindex[1] =
      interface_parent_element->vindex[0];
    interface_child_element->vindex[2] =
      interface_parent_element->vindex[1];
  }
  interface_child_element->descr =
    (ELEMENT2D_DESCRIPTION*)&helement_description;
  interface_child_element->present   = flags;
  interface_child_element->user_data = NULL;

  /* helement2d-entries */

  interface_child_element->parent = (HELEMENT2D*)interface_parent_element;
  if(flags & hefVinh){
    if(belement == interface_parent_element->belement->child[0]){
      for(i=0;i<number_of_vertices;i++)
        interface_child_element->vinh[i] = inheritance_rule_in_child_0[i];
    }
    else{
      for(i=0;i<number_of_vertices;i++)
        interface_child_element->vinh[i] = inheritance_rule_in_child_1[i];
    }
  }

  /* interface_element-entries */

  if(belement == interface_parent_element->belement->child[0]){
    interface_child_element->point_coords[0] =
      interface_parent_element->coord_of_new_node;
    interface_child_element->point_coords[1] =
      interface_parent_element->point_coords[2];
    interface_child_element->point_coords[2] =
      interface_parent_element->point_coords[0];
  }
  else{
    interface_child_element->point_coords[0] =
      interface_parent_element->coord_of_new_node;
    interface_child_element->point_coords[1] =
      interface_parent_element->point_coords[0];
    interface_child_element->point_coords[2] =
      interface_parent_element->point_coords[1];
  }

  for(i=0;i<3;i++){
    interface_child_element->coord_of_new_node[i] =
      0.5 * interface_child_element->point_coords[1][i] +
      0.5 * interface_child_element->point_coords[2][i];

    norm += interface_child_element->coord_of_new_node[i] *
            interface_child_element->coord_of_new_node[i];
  }

  norm = g_sqrt(norm) / 0.5;

  for(i=0;i<3;i++)
    interface_child_element->coord_of_new_node[i] /= norm;

  interface_child_element->belement = belement;
  interface_child_element->level    = interface_parent_element->level + 1;

  return;
}
```

Here like in `fill_interface_element_with_user_macro_element()` we have chosen a sphere geometry.

4.7 Helement Description

Remark: If you are only interested in the use of the standard GRAPE display methods on HMESHes (as to depict function values, extract isolines resp. isosurfaces, clip objects) you don't have to support the routines in the structure HELEMENT_DESCRIPTION. (for these methods the routines are not used at all)

element_world_to_coord()

For given world coordinates x = world[0], y = world[1] and z = world[2] the routine calculates the coordinates in the local coordinate system of the element. The result is written into `coord[]`.The return value is identical to that from `check_inside()`.

```
static int element_world_to_coord(HELEMENT2D* helement,
                                  double*     world,
                                  double*     coord)
{
  coord[0] = ...;
  coord[1] = ...;
  coord[2] = ...;

  return(helement->descr->check_inside(helement,coord));
}
```

In our example we have only triangles as elements and this routine reads:

```
static int element_world_to_coord(HELEMENT2D* helement,
                                  double*     world,
                                  double*     coord)
{
  double a[2][2];
  VEC2   b,c;
  int    i,j;

  ASSURE(helement,
         "hmesh2d: triangle_world_to_coord: no element!",
         return(0));

  ASSURE(helement->vertex,
         "hmesh2d: triangle_world_to_coord: no coordinates!",
         return(0));

  for(i=0;i<2;i++)
    for(j=0;j<2;j++)
      a[j][i] =  helement->vertex[i][j] - helement->vertex[2][j];

  for(j=0;j<2;j++)
    b[j] = world[j] - helement->vertex[2][j];

  if(!g_solve2(a,b,c)){
    fprintf(stderr,"hmesh2d-world-to-coord: point not in helement!\n");
    return(-2);
  }

  coord[0] = c[0];
  coord[1] = c[1];
  coord[2] = 1.0 - c[0] - c[1] ;

  return(helement->descr->check_inside(helement,coord));
}
```

element_coord_to_world()

For given coordinates in the local coordinate system x = coord[0], y = coord[1] and z = coord[2] of an element the routine calculates the coordinates in world coordinates. The result is written into `world[]`.

```
static void element_coord_to_world(HELEMENT2D* helement,
                                   double*     coord,
                                   double*     world)
{
  world[0] = ...;
  world[1] = ...;
  world[2] = ...;

  return;
}
```

For our case that translates to:

```
static void element_coord_to_world(HELEMENT2D* helement,
                                   double*     coord,
                                   double*     world)
{
  int i,j;

  ASSURE(helement,
         "hmesh2d: triangle_coord_to_world: no element!",
         return);

  ASSURE(helement->vertex,
         "hmesh2d: triangle_coord_to_world: no coordinates!",
         return);

  for(i=0;i<3;i++){
    world[i] = coord[0] * helement->vertex[0][i];

    for(j=1;j<3;j++)
      world[i] += coord[j] * helement->vertex[j][i];
  }

  return;
}
```

helement_description_check_inside()

returns `INSIDE` (-1), if the point with local coordinates `coord[]` is inside of the element's parameter range. Otherwise, at least one plane of a local face separates the element and the given point. In this case the local index of such a face is returned.

```
static int helement_description_check_inside(HELEMENT2D* helement,
                                             double*     coord)
{
  return_index = ...;

  return(return_index);
}
```

We could write it as:

```
static int helement_description_check_inside(HELEMENT2D* helement,
                                             double*     coord)
{
  double minimum      = -1.0E-5;
  int    return_index = -1;
  int    i;

  ASSURE(helement,
         "hmesh2d: helement_description_check_inside: no element!",
         return);

  for(i=0;i<3;i++){
    if(coord[i] < minimum){
      minimum      = coord[i];
      return_index = i;
    }
  }

  return(return_index);
}
```

helement_description_neighbour()

Returns a pointer to the adjacent element of the element across the edge with the given number at a point with local coordinates coord. After the call, coord contains the coordinates of the point transformed to the neighbouring element's local coordinate system. The structure helement2d may be overwritten.
coord = NULL is possible, then one of the neighbouring elements across the face will be returned.

flag :
EXACT_NEIGHBOUR -1 (coord has to be on the element boundary, then the neighbouring element with the coord point as a boundary point is returned)
FIRST_NEIGHBOUR 0 (return the first neighbour on this face)
NEXT_NEIGHBOUR 1 (succesively returns the list of neighbours)

```
static HELEMENT2D* helement_description_neighbour(HELEMENT2D*        helement,
                                                  int                edge,
                                                  int                flag,
                                                  double*            coord,
                                                  double*            world,
                                                  MESH_ELEMENT_FLAGS flags)
{
  INTERFACE_ELEMENT* interface_element;
  USER_ELEMENT*      user_element;
  USER_ELEMENT*      neighbour_element = NULL;

  ASSURE(helement,
         "hmesh2d: helement_description_neighbour: no element!",
         return(NULL));

  interface_element = (INTERFACE_ELEMENT*)helement;
  user_element      = interface_element->user_element;

  ...;                             /* get the appropriate neighbour */
  neighbour_element =  ...;        /* element, if possible.         */

  if(neighbour_element){
    ...;                           /* fill interface_element with the */
    ...;                           /* necessary data                  */

  /* remark: in the hierarchical setting some data on an element are obtained
             from the data on its parent element by inheritance;
             so to fill the interface_element with the appropriate
             neighbour_element data you have to supply the interface_element
             belonging to neighbour_element->parent too.
  */
  }
  else{
    ...;                           /* free the element */
  }

  return(helement);
}
```

helement_description_boundary()

Returns 0, if edge is an inner edge of the mesh, or a nonzero integer, if edge is part of the mesh's boundary.

```
static int helement_description_boundary(HELEMENT2D* helement,int edge)
{
  INTERFACE_ELEMENT* interface_element;
  USER_ELEMENT*      user_element;

  ASSURE(helement,
         "hmesh2d: helement_description_boundary: no element!",
         return(NULL));

  interface_element = (INTERFACE_ELEMENT*)helement;
  user_element      = interface_element->user_element;

  ...;                             /* see if there is a neighbour across */
                                   /* the edge                           */
  if(/* neighbour exists */)
    return 0;
  else
    return 1;
}
```

We modify this into:

```
static int helement_description_boundary(HELEMENT2D* helement,int edge)
{
  ASSURE(helement,
         "hmesh2d: helement_description_boundary: no element!",
         return(NULL));

  if(helement_description_neighbour(helement,edge,NULL,NULL,NULL))
    return 0;
  else
    return 1;
}
```

helement_description_coord_of_parent()

Given a point in an element by its local coordinates this routine calculates the local coordinates of this point in the parent element

```
static void helement_description_coord_of_parent(HELEMENT2D* helement,
                                                 double*     coord,
                                                 double*     parent_coord);
{
  int i;

  ASSURE(helement,
         "hmesh2d: helement_description_coord_of_parent: no element!",
         return(NULL));

  ...;

  for(i=0;i<3;i++)
    parent_coord[i] = ...;

  return;
}
```

For our BISMESH consisting of triangles we can formulate this like:

```
static void helement_description_coord_of_parent(HELEMENT2D* helement,
                                                 double*     coord,
                                                 double*     parent_coord);
{
  INTERFACE_ELEMENT* interface_element = (INTERFACE_ELEMENT*)helement;
  BELEMENT*          belement          = interface_element->belement;

  parent_coord[0] = coord[0];
  if(belement->node[1] == belement->parent->newnode){
    parent_coord[1] = .5 * coord[1];
    parent_coord[2] = 1 - parent_coord[0] - parent_coord[1];
  }
  else{
    parent_coord[2] = .5 * coord[2];
    parent_coord[1] = 1 - parent_coord[0] - parent_coord[2];
  }
  return;
}
```

4.8 Geometry Error Estimator

get_geometry_vertex_estimate()

This routine fills the array `results` with the error estimators of the vertices of the element.

```
static void get_geometry_vertex_estimate(HELEMENT2D* helement,
                                         double*     results,
                                         void*       geometry_data)
{
  int number_of_vertices = ...;
  int i;

  for(i=0;i<number_of_vertices;i++)
    results[i] = ...;

  return;
}
```

In our example we have a point based error estimator, which is calculated in a pre-roll step and stored in the array `geometry_data`.

```
static void get_geometry_vertex_estimate(HELEMENT2D* helement,
                                         double*     results,
                                         void*       geometry_data)
{
  int number_of_vertices = 3;
  int i;

  for(i=0;i<number_of_vertices;i++)
    results[i] = ((double*)geometry_data)[helement->vindex[i]];

  return;
}
```

get_geometry_element_estimate()

This routine returns the error estimator of the element.

```
static double get_geometry_element_estimate(HELEMENT2D* helement,
                                            void*       geometry_data)
{
  return ...;
}
```

The element estimator we choose is defined as the vertex estimator of the new node.

```
static double get_geometry_element_estimate(HELEMENT2D* helement,
                                            void*       geometry_data)
{
  BELEMENT* belement = ((INTERFACE_ELEMENT*)helement)->belement;

  return ((double*)geometry_data)[belement->newnode];
}
```

If different refinement rules with more new nodes on the next grid level are applied all these nodal error indicators have to be taken into account.

4.9 Routines for Functions on hmeshes

get_new_fhdata()

In analogy to `get_new_hmesh()` this routine generates a `f_hdata2d` structure. GRAPE uses `f_hdata2d` structures to access data on a function on a `hmesh2d`-object. In the interface mask this looks like:

```
F_HDATA2D* get_new_fhdata(...)
{
  F_HDATA2D* f_hdata2d = (F_HDATA2D*)mem_alloc(sizeof(F_HDATA2D));

  f_hdata2d->name                = "name of function";
  f_hdata2d->dimension_of_value  = ...;
  f_hdata2d->continuous_data     = ...;  /* 0 or 1 */
  f_hdata2d->f                   = data_access_function;
  f_hdata2d->f_el_info           = local_information;
  f_hdata2d->user_data            = ...;
  f_hdata2d->last                 = ...;  /* if there are other */
  f_hdata2d->next                 = ...;  /* functions          */
  f_hdata2d->function_data        = ...;
  f_hdata2d->get_bounds           = get_bounds;
  f_hdata2d->get_vertex_estimate  = get_function_vertex_estimate;
  f_hdata2d->get_element_estimate = get_function_element_estimate;
  f_hdata2d->threshold            = 0.0;
  f_hdata2d->hp_threshold         = 0.0;
  f_hdata2d->hp_maxlevel          = 0;

  return f_hdata2d;
}
```

In the sample program it has been adapted to:

```
F_HDATA2D* get_new_fhdata(double* function_values,double* error_estimates)
{
  F_HDATA2D* f_hdata2d = (F_HDATA2D*)mem_alloc(sizeof(F_HDATA2D));
  USER_FUNCTION_DATA* user_function_data;

  user_function_data =
    (USER_FUNCTION_DATA*)mem_alloc(sizeof(USER_FUNCTION_DATA));

  user_function_data->function_values = function_values;
  user_function_data->error_estimates = error_estimates;

  f_hdata2d->name                 = "name of function";
  f_hdata2d->dimension_of_value   = 1;
  f_hdata2d->continuous_data      = 1;  /* 0 or 1 */
  f_hdata2d->f                    = data_access_function;
  f_hdata2d->f_el_info            = local_information;
  f_hdata2d->user_data            = NULL;
  f_hdata2d->last                 = NULL;  /* if there are other */
  f_hdata2d->next                 = NULL;  /* functions          */
  f_hdata2d->function_data        = (void*)user_function_data;
  f_hdata2d->get_bounds           = get_bounds;
  f_hdata2d->get_vertex_estimate  = get_function_vertex_estimate;
  f_hdata2d->get_element_estimate = get_function_element_estimate;
  f_hdata2d->threshold            = 0.0;
  f_hdata2d->hp_threshold         = 0.0;
  f_hdata2d->hp_maxlevel          = 0;

  return f_hdata2d;
}
```

Here `USER_FUNCTION_DATA` is defined as:

```
typedef struct user_function_data {
  double* function_values;
  double* error_estimates;
} USER_FUNCTION_DATA;
```

After creating the f_hdata2d structure, you have to fill the hmesh with:

```
hmesh->f_data = (GENMESH_FDATA*)f_hdata2d;
```

If you want to handle more than one function on the hmesh, you create several structures of the type F_HDATA2D (function_1, function_2, ...) like above and add them to the hmesh by:

```
GRAPE (hmesh,"add-function")(&function_1);
GRAPE (hmesh,"add-function")(&function_2); ...
```

In our application we calculate the function values at the nodes and point based function error estimates in advance and store them in the `function_values` and `error_estimates` respectivly.

data_access_function()

The function specified by the pointer `f_hdata2d->f` is used to get the value(s) of a function at a given point. As can been seen above `f_hdata2d->f` points to the `data_access_function()`. The point for which the function value should be returned is specified either by the element and local coordinates or by the element and a local vertex index. The resulting function value is stored in the argument array `result`.

```
static void data_access_function(HELEMENT2D* helement,
                                 int         index,
                                 double      coord[],
                                 double      result[],
                                 void*       function_data)
{
  if(coord){
    ...;         /* calculate the function value at local coord
                    coord[0,..,max]
                 */
  }
  else{
    ...;         /* calculate the function value at
                    x = helement->vertex[i][0],
                    y = helement->vertex[i][1],
                    z = helement->vertex[i][2].
                 */
  }

  return;
}
```

As explained above we can access the nodal function values via the `function_data` argument. If `coord` is not `NULL` we apply a linear interpolation scheme.

```
static void data_access_function(HELEMENT2D* helement,
                                 int         index,
                                 double      coord[],
                                 double      result[],
                                 void*       function_data)
{
  double*             function_values;
  USER_FUNCTION_DATA* user_function_data;
  int i;

  user_function_data = (USER_FUNCTION_DATA*)function_data;
  function_values    = user_function_data->function_values;

  if(coord){
    *result = 0.0;
    for(i=0;i<3;i++)
      *result += coord[i] * function_values[helement->vindex[i]];
  }
  else
    *result = function_values[helement->vindex[index]];

  return;
}
```

local_information()

This routine provides element-dependent information about the function. Currently only the polynomial degree is supported.

```
static void local_information(HELEMENT2D*   helement,
                              F_HEL_INFO2D* result,
                              void*         function_data)
{
  result->polynomial_degree = ...;

  return;
}
```

We choose to have only a piecewise linear function on our mesh so the polynomial degree is always 1.

```
static void local_information(HELEMENT2D*   helement,
                              F_HEL_INFO2D* result,
                              void*         function_data)
{
  result->polynomial_degree = 1;

  return;
}
```

4.9.1 get_bounds()

This routine calculates the maximum and the minimum of the function on an element. In the mask we find:

```
static void get_bounds(HELEMENT2D* helement,
                double*     min,
                double*     max,
                void*       function_data)
{
  *min = ...;
  *max = ...;

  return;
}
```

And we adapt this to our piecewise linear function:

```
static void get_bounds(HELEMENT2D* helement,
                double*     min,
                double*     max,
                void*       function_data)
{
  USER_FUNCTION_DATA* user_function_data;
  double*             function_values;

  int i;

  user_function_data = (USER_FUNCTION_DATA*)function_data;
  function_values    = user_function_data->function_values;

  *min = function_values[helement->vindex[0]];
  *max = *min;

  for(i=0;i<2;i++){
    if(function_values[helement->vindex[i]] < *min)
      *min = function_values[helement->vindex[i]];
    if(function_values[helement->vindex[i]] > *max)
      *max = function_values[helement->vindex[i]];
  }

  return;
}
```

get_function_vertex_estimate()

The handling of function error estimators is analogous to the treatment of the geometry error estimators. We have stored the precalculated point estimators in the structure `f_hdata2d->function_data`. So this routine becomes:

```
static void get_function_vertex_estimate(HELEMENT2D* helement,
                                         double*     results,
                                         void*       function_data)
{
  USER_FUNCTION_DATA* user_function_data;
  double*             error_estimates;

  int number_of_vertices = 3;
  int i;

  user_function_data = (USER_FUNCTION_DATA*)function_data;
  error_estimates    = user_function_data->error_estimates;
  for(i=0;i<number_of_vertices;i++)
    results[i] = error_estimates[helement->vindex[i]];

  return;
}
```

get_function_element_estimate()

Again we have basically the same code as in `get_geometry_element_estimate`.

```
static double get_function_element_estimate(HELEMENT2D* helement,
                                            void*       function_data)
{
  USER_FUNCTION_DATA* user_function_data;
  double*             error_estimates;

  user_function_data = (USER_FUNCTION_DATA*)function_data;
  error_estimates    = user_function_data->error_estimates;

  return error_estimates[((INTERFACE_ELEMENT*)helement)->belement->newnode];
}
```

5 Fundamental Methods on Hierarchical Meshes

In this chapter the methods on procedural hierarchical interfaces as they are implemented in the software environment **GRAPE** are discussed in detail. This documentation only represents the current state of development. Continuously new methods are included in the library.

5.1 GenMesh

5.1.1 Memory Methods on GenMesh

(GENMESH *)GRAPE(GenMesh, "new-instance")(name)

CLASS *GenMesh char *name

This method creates a new instance of class `GenMesh` with name `name` by allocating the necessary memory. The `current_function`-pointer of the created `GenMesh` is initialized and the zero-function is set to be the actual function.

(GENMESH *)GRAPE(mesh, "free")()

GENMESH *mesh

This method frees the instance `mesh` by deleting the structures allocated by new-instance, that means the `GLIST *current_function`, the list of functions in `mesh`→`f_data` and the instance itself.

(GENMESH *)GRAPE(mesh, "softcopy")(copy)

GENMESH *mesh,*copy

Copies the instance `mesh` into `copy` by calling method on superclass and filling additional structure-pointers and variables, in particular the function-selector and the list of functions are copied. `copy` is returned.

(GENMESH *)GRAPE(mesh, "get-object")(obj,t)

GENMESH *mesh,*obj
double t

This method softcopies the instance `mesh` and its function-selector into the pointer `obj` and sets the time of `obj` to `t`. This `obj` is returned.

(GENMESH *)GRAPE(mesh, ”copy-functions”)(copy)

GENMESH *mesh,*copy

This method hardcopies the `f_data` list of the instance `mesh` to `copy`. If there are already some functions present in `copy`’s `f_data`-list, these are removed. The instance `mesh` is returned completely unchanged.

(GENMESH *)GRAPE(mesh, ”copy-function-selector”)(copy)

GENMESH *mesh,*copy

The function-selector of instance `copy` is deleted and the one of instance `mesh` is copied completely to `copy`. The instance `mesh` is returned unchanged.

(GENMESH *)GRAPE(mesh, ”clear-function-selector”)()

GENMESH *mesh

This method clears every slot in the list `current_function` of instance `mesh`.

5.1.2 Other Methods on GenMesh

GRAPE(mesh, ”xdr”)(xdr)

XDR *xdr

This method implements XDR interface of **GRAPE** for the class `GenMesh`. To use the read/write facilities of **GRAPE**, one has to overload this method.

(GLIST *)GRAPE(mesh, ”get-function-list”)()

GENMESH *mesh

The list `f_data` of instance `mesh` is copied into a `GLIST *` and this is returned. The method recognizes eventual occurance of circular lists and corrects it.

(GENMESH_FDATA *)GRAPE(mesh, ”get-function”)(create, preferred_slot, next_slot, ..., least_slot, NULL)

GENMESH *mesh
char *create, *preferred_slot, *next_slot, *least_slot

This method fulfils two tasks: First a new slot is created with name given by the string `create`, if this is not NULL and a slot with that name does not already exist. Second the slots of `mesh`’s function selector given by the following parameter-strings are inspected one by one. If a function has been selected for one of these slots, the data of this function is copied into the return parameter.

(GENMESH *)GRAPE(mesh, ”select-function”)(slot,function)

GENMESH *mesh
char *slot,*function

This method selects `function` for the given `slot`. If the `slot` does not exist, it is created.

(GENMESH *)GRAPE(mesh, "get-info")(list)

GENMESH *mesh
XLIST *list

Generates `list` with structure-dependent information of `mesh` like all `GenMesh`-entries and names of all functions in `mesh`'s `f_data`-list.

(GENMESH *)GRAPE(mesh, "get-access-mode")(flags)

GENMESH *mesh
MESH_ACCESS_FLAGS *flags

This method gets the actual `access_mode` entry of instance `mesh`. Valid values are combinations of predefined flags `mafNone`, `mafSorted` and `mafBoundary`. The result is returned in `flags`.

(GENMESH *)GRAPE(mesh, "set-access-mode")(flags)

GENMESH *mesh
MESH_ACCESS_FLAGS *flags

This method sets the parameter `flags` as actual `access_mode` entry of instance `mesh`. Valid values are combinations of predefined flags `mafNone`, `mafSorted` and `mafBoundary`.

(GENMESH *)GRAPE(mesh, "get-picked-point")(element,coord,point,key)

GENMESH *mesh
void **element
double *coord
VEC3 *point
int *key

This method allows picking a point either directly in the output-window by clicking left mouse-button or pressing 'p' while pointing on the output-window. In this case a layer for entering the point's coordinates pops up. This point is then searched in the given mesh by calling the appropriate search-method. All parameters are return-parameters: `point` contains the world-coordinates of the picked point, `key` contains 1001 if left mouse-button was pressed, 1002 or 1003 for other mouse-buttons and 'p' if this key was pressed. The element that contains the picked point and its local coordinates are returned in `element` and `coord`.

5.2 Mesh2d

5.2.1 Memory Methods on Mesh2d

(MESH2D *)GRAPE(Mesh2d, "new-instance")(name)

CLASS *Mesh2d
char *name

This method creates a new instance of class `Mesh2d` with name `name` and allocates the necessary memory.

(MESH2D *)GRAPE(mesh, "free")()

MESH2D *mesh

Just calls identical method on superclass `GenMesh`.

(MESH2D *)GRAPE(mesh, "softcopy")(copy)

MESH2D *mesh,*copy

Copies the instance `mesh` into `copy` by calling the method on the superclass and additionally fills structure-pointers in `copy`, `copy` is returned.

(MESH2D *)GRAPE(mesh, "add-function")(f_data)

MESH2D *mesh
F_DATA2D *f_data

Adds the function given by `f_data` by copying and inserting it as first entry in the linked list given by `mesh`→`f_data`. Finally the function-selector is updated.

(MESH2D *)GRAPE(mesh, "remove-function")(name)

MESH2D *mesh
char *name

This method frees the first entry of the linked list `mesh`→`f_data` which corresponds to the given string `name`. The function-selector is updated afterwards.

5.2.2 Display Methods on Mesh2d

(MESH2D *)GRAPE(mesh, "display")()

MESH2D *mesh

The `Mesh2d` instance `mesh` is rendered on the standard device in a default manner. In patch and texture mode the elements are displayed as flat shaded patches, in grid mode the edges of the elements are drawn.

(MESH2D *)GRAPE(mesh, "isoline")(select, highscal, colorbar)

(MESH2D *)GRAPE(mesh, "isoline-disp")()

(MESH2D *)GRAPE(mesh, "isoline-select-disp")()

MESH2D *mesh
int select
double highscal
COLORBAR* colorbar

These two display-methods draw isolines and iso-patches on the `mesh`. For having nearly the same functionality the drawing is done in both cases by one method `"isolines"` just differing in the flag `select`: `select`=0 means call from `"isoline-disp"`, `select`=1 indicates call from `"isoline-select-disp"`. In grid-mode `"isoline-disp"` draws equidistant isolines with respect to the `colorbar`, that means line-distance, colors and min/max-values are taken from `colorbar`. `"isoline-select"` just draws selected isolines. In patch-mode iso-patches are drawn, the select-display version additionally draws the selected lines. Texture mode offers the same functionality as the `HMesh2d`-version, see 5.3.2 for details. All modes additionally allow to extrude the mesh in z-direction according to the function-values. By this a simultaneous function-graph/isoline-display can be obtained.

(MESH2D *)GRAPE(mesh, ”vect”)(double length, double width, double dist, double color_scal, double offset_scal, double min, double max)

(MESH2D *)GRAPE(mesh, ”vect-disp”)()

MESH2D *mesh

These methods offer the same functionality as the Fe2d-analogon.

(MESH2D *)GRAPE(mesh, ”function-graph-disp”)()

MESH2D *mesh

This method draws the instance `mesh` not as a plain mesh but the nodal z-coordinates are shifted by a multiple of the nodal function-values. The factor of this multiple can be changed in the options-layer for this display-method.

(MESH2D *)GRAPE(mesh, ”geometry-graph-disp”)()

MESH2D *mesh

Similar to the `"isoline-disp"` method, this method also draws isolines/iso-patches on the `mesh`. Instead of corresponding to the current function, these isolines belong to the z-coordinate-function of the mesh. Thus real geometry-isolines are drawn. This method is faster than using the function-selector and choosing the function z-coordinate, which gives the same result.

(MESH2D *)GRAPE(mesh, ”shrink-disp”)()

MESH2D *mesh

Similar to the standard display-method this one draws each element by a shrinked patch or shrinked polygon depending on the grid-patch mode. The shrink-factor can be changed by a corresponding ruler.

5.2.3 Other Methods on Mesh2d

GRAPE(mesh, ”xdr”)(xdr)

XDR ∗xdr

This method implements XDR interface of **GRAPE** for the class `Mesh2d`. Of course neither function pointers nor the underlying user data can be handled, but all scalar values are converted, so it is an abstract method. To use the read/write facilities of **GRAPE**, one has to overload this method.

(MESH2D *)GRAPE(mesh, "get-info")(list)

MESH2D *mesh
XLIST *list

Fills `list` with structure-dependent information-strings. Actually it calls `"GenMesh-get-info"` and adds the information `mesh→dimension_of_world`.

(GLIST *)GRAPE(mesh, "get-function-list")()

MESH2D *mesh

Generates list of functions including default-functions.

(MESH2D *)GRAPE(mesh, "get-min-max")(pmin,pmax)

MESH2D *mesh
double *pmin,pmax

Evaluates the current data-function in all nodes of the `mesh` and determines the extremal values of these.

(MESH2D *)GRAPE(mesh, "value-min-max-send")()

MESH2D *mesh

This method calls `"get-min-max"` and prints the values to `stdout`.

(MESH2D *)GRAPE(mesh, "get-bnd-box")(xmin,xmax,ymin,ymax,zmin,zmax)

MESH2D *mesh
double *xmin,*xmax,*ymin,*ymax,*zmin,*zmax

Determines the bounding-box of the instance `mesh` and returns it by its extremal coordinates.

(MESH2D *)GRAPE(mesh, "search")(v,c,el)

MESH2D *mesh
VEC3 v
double *c
ELEMENT2D **el

This method searches within a `mesh` for a given point with world-coordinates `v`. The search applies neighbourhood-relationship if this is provided by an `element→descr→neighbour` function. Otherwise it uses straightforward mesh-traversal. If successful, the method returns in `el` the element which contains the given point and in `c` the local coordinates of this point. The pointer `c` has to be allocated with enough memory before calling the method. If the search failed, `el` will be NULL.

(MESH2D *)GRAPE(mesh, "find-pick-intersec")(v,u,w,el,c)

MESH2D *mesh
VEC3 v
double u,w
ELEMENT2D **el
double *c

For a mesh with `dimension_of_world` equal 2 this method determines for given screen-coordinates `u`,`w` which point on the mesh is hit by these pick-coordinates. The world coordinates of this point are returned in `v`, the element which contains this point and its local-coordinates are returned in `el` and `c`.

5.3 HMesh2d

5.3.1 Memory Methods on HMesh2d

(HMESH2D *)GRAPE(HMesh2d, "new-instance")(name)

CLASS *HMesh2d char *name

This method creates a new instance of class `HMesh2d` with name `name` and allocates the necessary memory. The function pointers `first_element` and `next_element` are set to default procedures.

(MESH2D *)GRAPE(hmesh, "softcopy")(copy)

HMESH2D *hmesh,*copy

Copies the instance `hmesh` by calling the method on the superclass `Mesh2d` and fills additional structure-pointers in `copy`. `copy` is returned.

(HMESH2D *)GRAPE(hmesh, "add-function")(f_hdata)

HMESH2D *hmesh
F_HDATA2D *f_hdata

Adds the function given by `f_hdata` by copying and inserting it as first entry in the linked list given by `hmesh→f_data`. Finally the function-selector is updated.

(HMESH2D *)GRAPE(hmesh, "remove-function")(name)

HMESH2D *hmesh
char *name

This method frees the first entry of the linked list `hmesh→f_data` which corresponds to the given string `name`. The function-selector is updated afterwards.

(HMESH2D *)GRAPE(hmesh, "get-object")(obj,t)

HMESH2D *hmesh,*obj
double t

This method returns the instance `hmesh` at time `t`. This happens by softcopying `hmesh` and its function-selector to the pointer `obj`, setting `obj`'s time to `t` and returning `obj`. If `obj` already is filled, its `level_of_interest` and `threshold` are kept. (To be precise: `hmesh`'s parameters are set to these values.)

(HMESH2D *)GRAPE(hmesh, "hmesh-interactive-send")()

HMESH2D *hmesh

This method sets up the project *uif-hm2*, see 5.4.

(HMESH2D *)GRAPE(hmesh, "calc-lenstimators")(lens)

> **HMESH2D *hmesh**
> **HMESH2D_LENS *lens**

This method calculates lens-estimators for all nodes of the `hmesh` up to the leaf-level and returning them in `lens`→`lens_data`. The calculation of the estimators is based on the distance of parent-nodes to child-nodes. These estimators are saturated by a blow-up procedure assuring that estimators of nodes of an element are bigger than all nodes of the elements's children. It is necessary that elements have access to their global vertex-indices in `vindex`. If this is not the case all estimators are set to zero.

5.3.2 Display Methods on HMesh2d

All display-methods on HMeshes offer the possibility of adaptive visualization with regard to various thresholds and error estimates which have to be provided by the user in a prescribed way or can also for example in case of lens-estimators be calculated by default-routines. See the example-HMesh-interfaces for further details. Without any error-estimates the HMesh is always drawn up to the level given by its `level_of_interest` entry, whereas with error-estimates some elements may be extracted on a higher hierarchy-level without losing essential detail-information but gaining a speedup because of this partial inspection of the mesh.

(HMESH2D *)GRAPE(hmesh, "display")()

> **HMESH2D *hmesh**

The `HMesh2d` instance `hmesh` is rendered on the standard device in a default manner. In patch and texture mode the elements are displayed as flat shaded patches, in grid mode the edges of the elements are drawn. If geometry-estimators or lens-estimators are provided, these are used for adaptive visualization. An options-layer allows to activate/deactivate the lens-adaptivity.

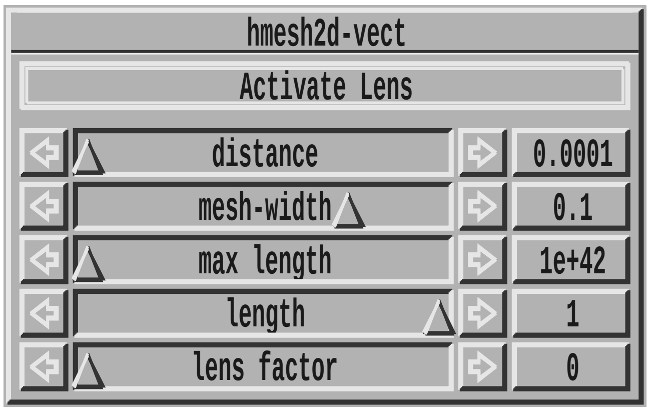
(HMESH2D *)GRAPE(hmesh, "vect-disp")()

> **HMESH2D *hmesh**

This method draws a 2d or 3d vector-valued function as arrow-field on the mesh. The mesh is drawn in a default manner, the arrows as flat arrows in 3d-space starting from points of a virtual equidistant grid put on the instance `hmesh`.
The options-layer shown in Figure 4 allows changing diplay-parameters.
The ruler `distance` determines the z-shift of all arrows, the `mesh-width` is the distance between the arrows. `Length` gives a scale-factor which is applied to each function-value-vector. If the resulting length is bigger than the value `max length`, the vector is shortened to this value. If lens-estimators are provided, a lens can be activated/deactivated and a final ruler called `lens factor` allows to enlargen the mesh-width automatically in case of activated lens. An example is shown in figure 5.

(HMESH2D *)GRAPE(hmesh, "rgb-disp")()

> **HMESH2D *hmesh**


hmesh2d-vect

Activate Lens

distance	0.0001
mesh-width	0.1
max length	1e+42
length	1
lens factor	0

Figure 4: Options `Layer` for HMesh2d-vect-disp

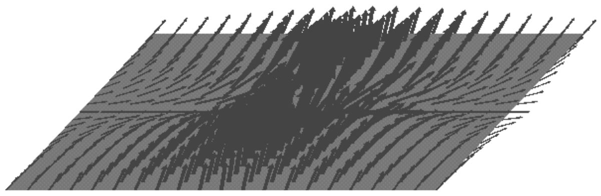


Figure 5: example of `"vect-disp"`

This method draws vector-valued functions encoding the data in rgb-space. This is done by cutting the function-vectors componentwise to the unit-interval and taking these values as rgb-vector. Every element is patched with these colors in its vertices. A lens is automatically initialized based on the estimates generated by the method `"calc-lenstimators"`. Grid-mode is not supported. An example is shown in Figure 6.

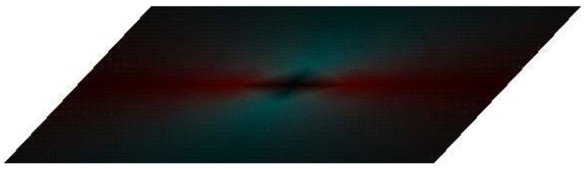


Figure 6: Example of `"rgb-disp"`

(HMESH2D *)GRAPE(hmesh, ”shrink-disp”)()

HMESH2D *hmesh

Similar to the standard display-method this one draws each element by a shrinked patch or shrinked polygon depending on `grid-patch` mode. The shrink-factor can be changed by a corresponding ruler. Additional a lens can be activated/deactivated. An example is shown in Figure 7.

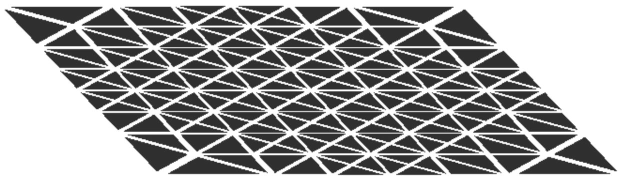


Figure 7: example of `"shrink-disp"`

(HMESH2D *)GRAPE(hmesh, "geometry-graph-disp")()

HMESH2D *hmesh

Similar to the `"isoline-disp"` method, this method also draws isolines/isopatches on the `hmesh`. Instead of corresponding to the current function, these isolines belong to the z-coordinate-function of the `hmesh`. Thus real geometry-isolines are drawn. This method is faster than using the function-selector and choosing the function z-coordinate, which gives the same result. With the options-layer shown in Figure 8 a lens can be activated/deactivated, the display can be switched to a projection on the x/y-plane and the colorbar can be accessed. An example is shown in Figure 9.

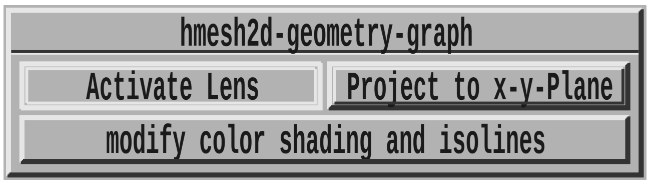


Figure 8: Options `Layer` for HMesh2d-geometry-graph-disp

(HMESH2D *)GRAPE(hmesh, "function-graph-disp")()

HMESH2D *hmesh

This method draws the instance `hmesh` not as a plain mesh but the nodal z-coordinates are shifted by a multiple of the nodal function-values. The factor of this multiple can be changed as `graph-height` in the options-layer for this display-method shown in figure 10. A lens can

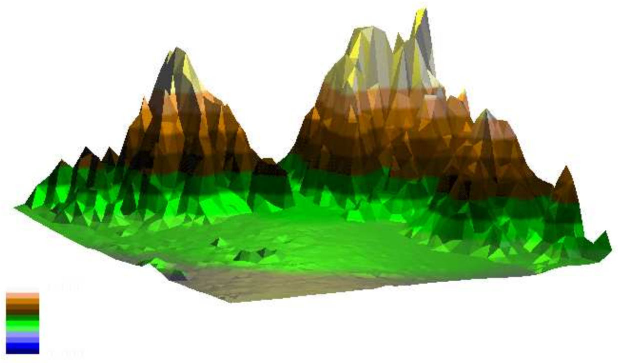


Figure 9: example of "geometry-graph-disp"

be activated/deactivated if lens-estimators are supported in the instance **hmesh**. An example is shown in figure 11.

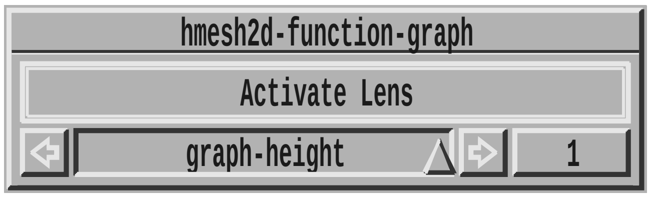


Figure 10: Options `Layer` for HMesh2d-geometry-graph-disp

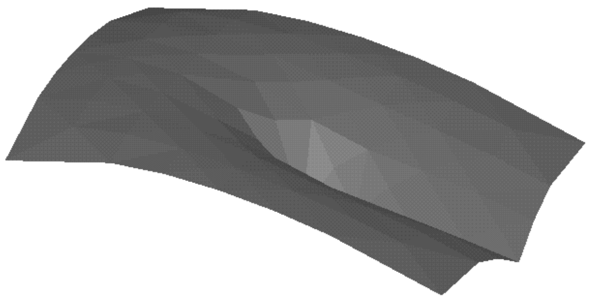


Figure 11: example of "function-graph-disp"

(HMESH2D *)GRAPE(hmesh, "isoline-disp")()

(HMESH2D *)GRAPE(hmesh, "isoline-select-disp")()

HMESH2D *hmesh

These two display-methods draw isolines and iso-patches on the mesh. In grid-mode "`isoline-disp`" draws equidistant isolines with respect to the actual colorbar, that means line-distance, colors and min/max-values are taken from the colorbar. `"isoline-select-disp"` just draws selected isolines. In patch-mode iso-patches are drawn, the select-display version additionally draws the selected lines. The following figure shows the options-layer for this method in texture mode. In other modes some buttons/rulers are hidden, if they are not used. In

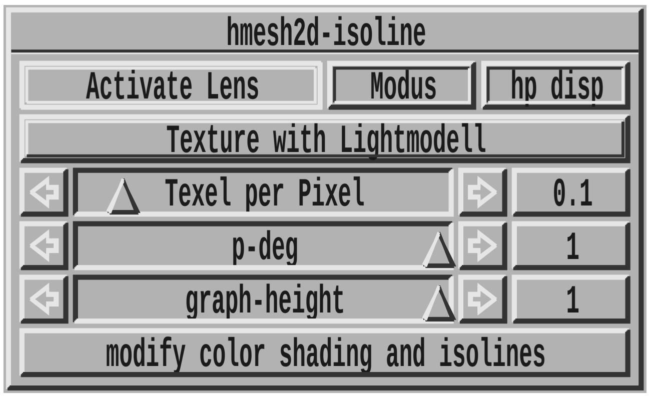


Figure 12: Options `Layer` for HMesh2d-isoline-disp

texture-mode the surface of the mesh is patched with textures which are able to react on light-sources. This ability can be activated/deactivated by the button `Texture with lightmodel`. The button `modus` activates/deactivates the ruler `graph-height` and an additional extruding of the mesh in z-direction according to a multiple of the nodal function-values, the factor given by `graph-height`. The button `hp disp` activates the remaining two rulers `Texel per Pixel` and `p-deg`. This mode draws higher polynomial textures on each element. The polynomial degree and resolution of the textures are determined by these rulers. An example of this display-method is shown in Figure 13.

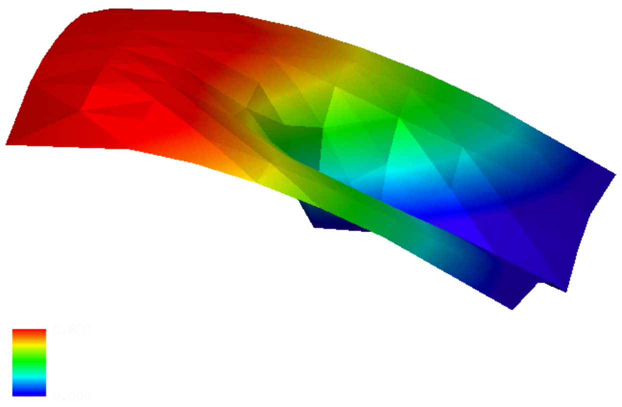


Figure 13: example of `"isoline-disp"`

(HMESH2D *)GRAPE(hmesh, ”debug-disp”)()

HMESH2D *hmesh

This method is a debug-tool for hmesh-interfaces. It displays the `hmesh` like the shrink-method and the element’s boundaries by blue lines. Therefore the shrink-options-layer is shown. The traversal-functions of the hmesh-interface can now be called by pressing the corresponding buttons of the options-layer for this method shown in Figure 14.

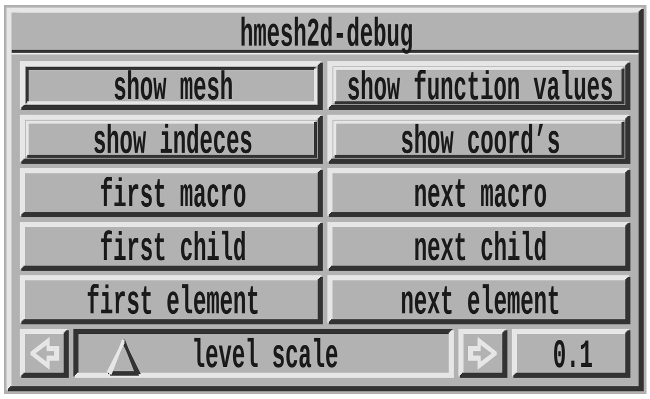


Figure 14: Options `Layer` for HMesh2d-debug-disp

The resulting element and its parents-path to a macro-element is highlighted by white boundaries. Child-elements are shifted from their parents in z-direction according to the value of the ruler `level-scale`. Additional information is shown for the actual element by activating one of the buttons `show indices`, `show coord's` or `show function-values`. The drawing of the mesh can be enabled/disabled by the button `show mesh`. An example is shown in Figure 15.

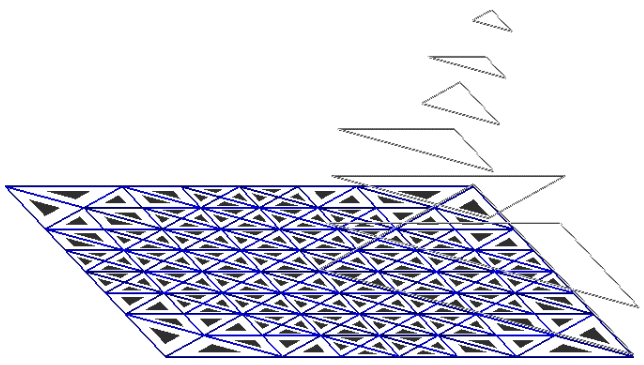


Figure 15: example of `"debug-disp"`

5.3.3 Other Methods on HMesh2d

GRAPE(hmesh, ”xdr”)(xdr)

XDR *xdr

This method implements XDR interface of **GRAPE** for the class `HMesh2d`. Of course neither function pointers nor the underlying user data can be handled, but all scalar values are converted, so it is an abstract method. To use the read/write facilities of **GRAPE**, one has to overload this method.

(HMESH2D *)GRAPE(hmesh, "get-bnd-box")(xmin, xmax, ymin, ymax, zmin, zmax)

HMESH2D *hmesh
double *xmin,*xmax,*ymin,*ymax,*zmin,*zmax

Determines the bounding-box of the instance `mesh` and returns it by its extremal coordinates. This method is faster than the `Mesh2d`-version because only macro-elements have to be inspected.

(HMESH2D *)GRAPE(hmesh, "search")(v, c, el)

HMESH2D *hmesh
VEC3 v
double *c
HELEMENT2D **el

This method searches within the `hmesh` for a given point with world-coordinates `v`. If `el` is not NULL local search is performed starting at the element `el`. This means walking up in the hierarchy until an element is found, which contains `v`, and then walking down to the wanted element. If `el` is NULL at the beginning, hierarchical search is applied by walking over the macro-elements and then walking down the hierarchy.
If successful, the method returns in `el` the element on level `level_of_interest` which contains the given point `v`, and in `c` the local coordinates of this point. The pointer varc has to be allocated with enough memory before calling the method. If the search failed, `el` will be NULL. The search relies on the `HMesh2d`-method `"select_child"`. On one hand this means that this pointer has to be filled in the interface at least with the default-method `"g_hmesh2d_def_select_child"`. On the other hand it implies that implementing this method efficiently is a possibility of speeding up the search.

(HMESH2D *)GRAPE(hmesh, "find-pick-intersec")(v,u,w,el,c)

HMESH2D *hmesh
VEC3 v
double u,w
HELEMENT2D **el
double *c

For `hmesh` with `dimension_of_world` equal 2 this method determines for given screen-coordinates `u`,`w` which point on the mesh is hit by these pick-coordinates. The world coordinates of this point are returned in `v`, the element on level `mesh`$\rightarrow$`level_of_interest` which contains this point and its local-coordinates are returned in `el` and `c`. The only difference to the `Mesh2d`-version is the use of improved hierarchical searching.

5.4 Projects for Meshes

All meshes can be accessed interactively by the projects *uif-gm*, *uif-hm2* and *uif-hm3*. The first of these projects generates an options layer as in Figure 16. It is an interface for the function-selector of the mesh. The leftmost button pops up a menu to select the slot, according to this the selected function automatically is displayed on the right button. This button also allows

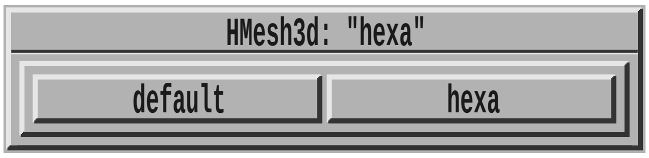


Figure 16: `Layer` for Project *uif-gm*

to change the selected function for the given slot by pressing it. The list of possible choices popping up after pressing contains the user-defined ones and additional some default-functions like coordinate-functions.
The project *uif-hm2* generates the same layer, but additional a second one to change the entries of a `HMesh2d` instance like `level-of-interest` and various threshold-values like `geometry-threshold`, `threshold` and `hp-threshold` for adaptive visualization with regard to the geometry, the function-values and polynomial-degree. This layer is shown in Figure 17.

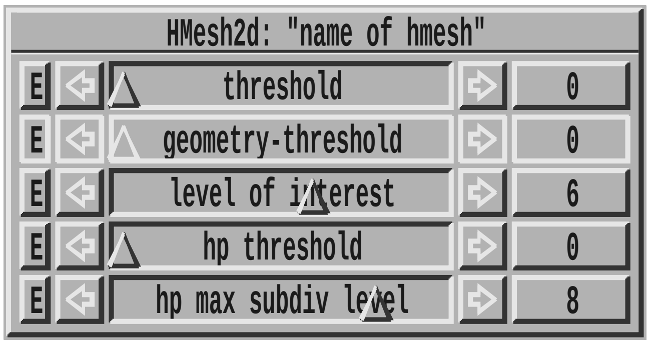


Figure 17: Additional `Layer` for Project *uif-hm2*

5.5 Methods on other classes related to meshes

(HMESH2D_LENS *)GRAPE(lens, ”get-lens”)(name)

HMESH2D_LENS *lens
char *name

This method creates a new instance of Class `Hmesh2d_Lens` with name given as parameter `name` and initializes it with default values. In particular the estimators and thresholds are set to default-functions. The lens is assigned to an own interactive layer which allows to manipulate the lens-parameters.

(HMESH2D_LENS *)GRAPE(lens, ”display”)()

HMESH2D_LENS *lens

This method draws an icosaeder related to the parameters of the `lens` as a wire-frame-model. The center is the entry `coord` of the instance `lens`. The radius is a convex-combination depending on `show_threshold` between the inner and outer region of the mesh.

(MESH2D *)GRAPE(sc, ”convert-to-mesh-send”)()

SCENE *sc

This method tries to convert the instance `sc`→`object` to one of class `Mesh2d`. If successful, this new instance is taken as `sc`'s object. The old structure is still accessible via the created mesh's `user_data` pointer.

(MESH2D *)GRAPE(tr, "convert-to-mesh2d")()

TRIANG2D *tr

This method generates a `Mesh2d` instance based on the data of `tr`. Especially the original `Triang2d` structure remains unchanged and is accessible from the returned mesh via its `user_data` pointer. The resulting mesh carries no functions.

(MESH2D *)GRAPE(fe, "convert-to-mesh2d")()

FE2D *fe

This method generates a `Mesh2d` instance based on the data of `fe`. Especially the original `Fe2d` structure remains unchanged and is accessible from the returned mesh via its `user_data` pointer.

5.6 HMesh3d

5.6.1 Memory Methods on `HMesh3d`

(HMESH3D *)GRAPE(HMesh3d, "new-instance")(name)

CLASS *HMesh3d char *name

This method creates a new instance of class HMesh3d with name "name" and allocates the necessary memory. The function pointers `first_element` and `next_element` are set to default procedures.

GRAPE(hmesh, "free")()

HMESH3D *hmesh;

The associated memory of the instance variables is freed and the instance is deleted.

GRAPE(hmesh, "softcopy")(copy)

HMESH3D *hmesh;
HMESH3D *copy;

Make a softcopy of a HMesh3d instance. If copy is not NULL then this instance is used to store the copy otherwise a new HMesh3d instance is created.

(HMESH3D *)GRAPE(hmesh, "add-function")(f_hdata)

HMESH3D *hmesh
F_HDATA3D *f_hdata

Adds the function given by `f_hdata` by copying and inserting it as first entry in the linked list given by `hmesh`→`f_data`. Finally the function-selector is updated.

(HMESH3D *)GRAPE(hmesh, "remove-function")(name)

HMESH3D *hmesh
char *name

This method frees the first entry of the linked list `hmesh`→`f_data` which corresponds to the given string `name`. The function-selector is updated afterwards.

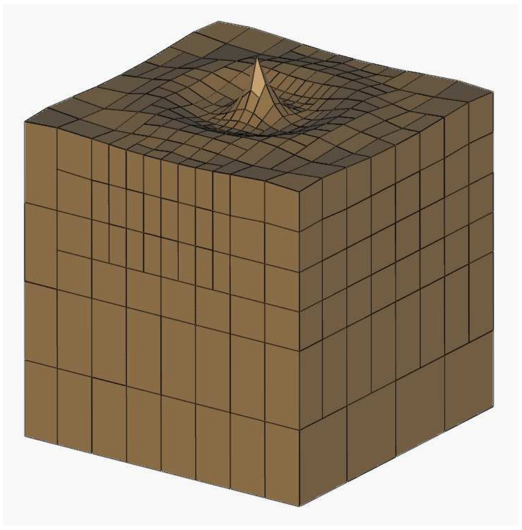


Figure 18: Adaptive visualization of the boundary of a domain.

(HMESH3D *)GRAPE(hmesh, "get-object")(obj,t)

HMESH3D *hmesh,*obj
double t

This method returns the instance `hmesh` at time `t`. This happens by softcopying `hmesh` and its function-selector to the pointer `obj`, setting `obj`'s time to `t` and returning `obj`. If `obj` already is filled, its `level_of_interest` and `threshold` are kept. (To be precise: `hmesh`'s parameters are set to these values.)

5.6.2 Display Methods on `HMesh3d`

All display-methods on HMeshes offer the possibility of adaptive visualization with regard to various thresholds and error estimates which have to be provided by the user in a prescribed way or can be calculated by default-routines. See the example-HMesh-interfaces for further details. Without any error-estimates the HMesh is always drawn up to the level given by its `level_of_interest` entry, whereas with error-estimates some elements may be extracted on a higher hierarchy-level without losing essential detail-information but gaining a speedup because of this partial inspection of the mesh.

(HMESH3D *)GRAPE(hmesh, "display")()

HMESH3D *hmesh

The HMesh3d instance hmesh is rendered on the standard device in a default manner. All the boundary faces of the elements are displayed as flat shaded patches in patch mode and in grid mode the edges of these faces are drawn. If boundary-estimators are provided, these are used for adaptive visualization.

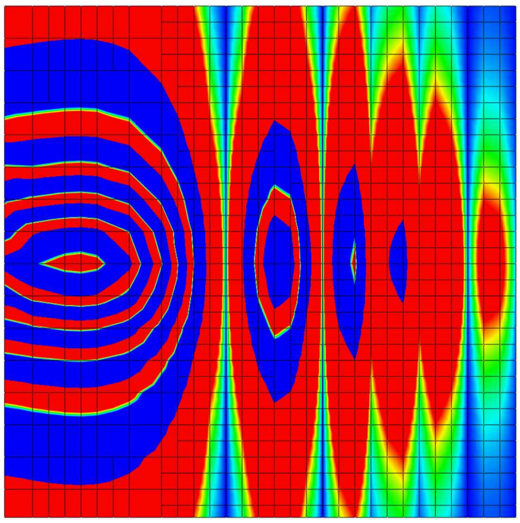


Figure 19: Colorshading on a slice.

(HMESH3D *)GRAPE(hmesh, ”clip”)(f, var, draw_clip, scal)

GRAPE(mesh, ”clip-ball-disp”)()

GRAPE(mesh, ”clip-plane-disp”)()

GRAPE(mesh, ”clip-deg-fine-disp”)()

GRAPE(mesh, ”clip-isoline-disp”)()

GRAPE(mesh, ”clip-vect-disp”)()

```
HMESH3D *hmesh
MESH3D *mesh
double (*f)(VEC3 x,void *var)
void *var
int (*draw_clip)(CLIPM3D_PAR)
void *scal
```

These methods are designed to support general clipping facilities with varying hyper surface and local rendering function. The most basic method with the greatest flexibility is the method ”clip”. The others are special predefined interactive methods derived from the latter one. The domain of the mesh is split by the implicitly given clipping function f into a visible ($f > 0$) and an invisible ($f < 0$) part. The zero level of this clipping function defines the intersection surface. The method ”clip” on HMesh3d supports hierarchical search algorithms for extracting the intersection surface and uses adaptive resolution strategies, if the error estimator of the underlying function data is supplied.

The variable *var* in the parameter list of the method call and of this function *f* must be of the same type. The parameter **var* allows an interactive varying of the clipping function. The implicit functions describing a plane or a ball are predefined:

typedef struct MPLANE_PARM = { double n_x, n_y, n_z, distance;};

*double mplane(VEC3 x, MPLANE_PARM *var)*

typedef struct MBALL_PARM = { double cx,cy,cz,radius;};

*double mball(VEC3 x, MBALL_PARM *var)*

The pointer *draw_clip* points to a function which renders the surface of intersection on a single element. This function works on the data corresponding to one single clipped element. A NIL pointer causes the standard visualization of the surfaces including normal shading.
There are some predefined local rendering functions:

- **draw_clipm_isoline:** generates an isoline image of a one dimensional FEM function or of the absolute value of a vector–valued function.
- **draw_clipm_deg_fine:** shows the size of each clipped element in colour.
- **draw_clipm_vect:** draws a mesh of vector valued function values projected onto the tangent plane of the intersection surface, where the colour represents the component normal the tangent plane (only applicable if the function is three dimensional).

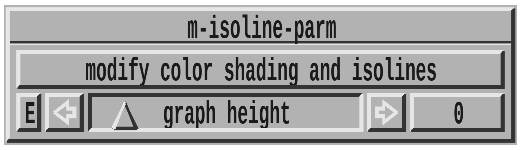


Figure 20: Interactive layer of the "clip-isoline-disp" method.

The interactive methods "clip-isoline-disp", "clip-deg-fine-disp", "clip-vect-disp" use these routines while calling the standard clipping method "clip". Figure 20 shows the interactive layer of the "clip-isoline-disp" method. With the rulers "normal x/y/z" the normal of a clipping plane is chosen, whereas the "distance" ruler adjusts the plane to the desired position. The ruler "graph height" gives a scaled height to the graph of the displayed isolines. Finally the "clolorbar" interface can be used to adjust the colours of the isolines to the given function values.
To construct a new private rendering method working on the surface of intersection, the user can add new clipping methods by using other and different surfaces or by adding new local rendering functions. Therefore we now explain the parameter structure for the local rendering function **draw_clip**. The general 'clip' method calls this function in the following way:

draw_clip((CLIPM3D_PAR *clip3d_par)

The structure clip3d_par with the following typedef is filled by the general "clip" method.

```
typedef struct clipm3d_par
{ ELEMENT3D *e;
  int n;
  VEC3 *v;
  double *coord;
  double (*f)();
  void *var;
  void *scal;
  int flag ;
  int dimension_of_value;
} CLIPM3D_PAR;
```

This structure describes the intersection of the hyper surface with a single element. The filled structure is delivered by the actual method "clip". The variable e points to the intersected element and n gives the number of intersection points of one connected intersection patch. The array $v[0 \cdots n-1][0 \cdots 2]$ contains the global Euclidean coordinates of these points. They are located on edges of the element's boundary at a linearly interpolated intersection position. In all the listed interactive methods a plane is used as the clipping function. *coord* is a pointer to a list of the local coordinates of the intersection points , which are stored in global coordinates in v. In this 1D array the i'th coordinate vector starts at position $(coord + i * d)$ where d is the dimension of coordinate space on the current element. The pointer *var* points to the parameters of the function describing the intersecting hyper surface. At the memory address *scal* the function *draw_clip* finds a user defined structure filled with parameters for the specific rendering style. These parameters can be influenced interactively, if the user supports this in an appropriate interactive method (see also the GRAPE reference manual).

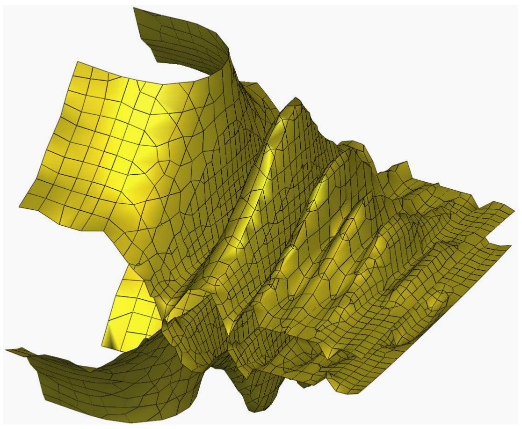


Figure 21: Adaptive visualization of an iso–surface.

(HMESH3D *)GRAPE(hmesh, "level")(func, lev, mode)

GRAPE(mesh, "level-disp")()

> **HMESH3D *hmesh**
> **MESH3D *mesh**
> **void (*func)(ELEMENT3D *el, int, double *coord, double *val, void *function_data)**
> **double lev**
> **int mode**

The slices of those elements where the one-dimensional finite element function *func* has the level *lev* are calculated and displayed. The parameters of *func* are equal to those used for a one dimensional FEM function on a *HMesh3d* instance: *f(HELEMENT3D *el, double *coord, double *val, void *function_data)*. If *func* is NULL and *hmesh* includes a function then this finite element function is taken into account. If its dimension of value is greater than 1 the absolute value of the value vector defines the function under consideration. The resulting 2D level surface consists of a polygonal approximation. If "mode = 1" or "mode = -1" the super or sub volumes of the function on the volume geometry are shown.

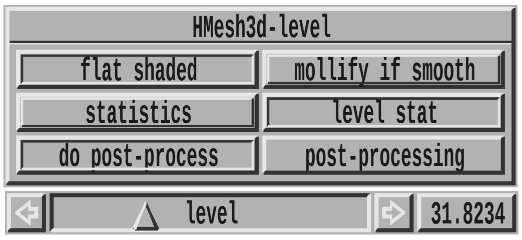


Figure 22: Interactive layer and ruler of the "level-disp" method.

The method "level-disp" calls the method "level" with the standard finite element function and inserts a ruler (cf. Figure 22) into the options menu to adjust the value "lev" interactively. The method "level" on HMesh3d supports hierarchical and adaptive search algorithms, if the error estimators of the underlying function data is supplied. Furthermore an interactive layer with the following buttons is added:

- **flat shaded** Turns on/off the flat shading mode.
- **mollify if smooth** If the flat shading mode is turned off the extracted grid of the isosurface is smoothed by an mollify algorithm.
- **statistics** Prints the threshold value and the number of actually rendered elements into the graphic window. Three rulers to adjust the position of the information in the graphic window appear.
- **level stat** Writes some information about the hierarchical and adaptive extraction of the isosurface onto the standard output.
- **do post-process** Calls the routine "g_PostProcessCurrSurfMesh" if "flat shaded" is turned off. If this routine is selected the extracted surface mesh is smoothed and small cracks in the surface are closed. Such cracks may occure in the case of hexahedrons, where data is interpolated trilinear.
- **post-processing** Calls the method "post-processing-send", which is described below.

(HMESH3D *)GRAPE(hmesh, "post-processing-send")()

HMESH3D *hmesh

The method "post-processing-send" inserts a post processing layer into the options menu, which enables the user to activate one or more cutting planes, which cut off certain areas of the isosurface (cf. Figure 23). In detail there is the following functionality:

- **edit** Inserts the interfaces of the actual cutting planes.
- **add plane** Adds a cutting plane. An interface to adjust the position of the plane appears.

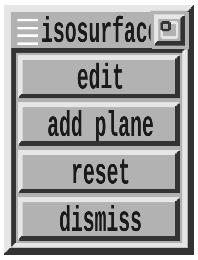


Figure 23: The interface inserted by the method "post-processing-send".

- **reset** Resets to a default status.
- **dismiss** Closes the interactive interface.

GRAPE(hmesh,"shrink-disp")()

HMESH3D* hmesh

This method draws all elements of the mesh shrinked by a factor which can be changed by the corresponding ruler. The elements are drawn as a grid or as patches depending on the `grid-patch` mode. The method uses the `threshold` and `get_element_estimate` of the meshs `f_data` to determine whether it displays elements on a coarser level.

(HMESH3D*) GRAPE(hmesh,"inspect-disp")()

HMESH3D* hmesh

The `"inspect-disp"` method works similar to its twodimensional pendant. It displays the complete mesh with shrinked elements (as grid or as patches) and a grid of the boundary-faces of the mesh using blue lines. It is possible to traverse the mesh hierarchically via the buttons of the group `select element to inspect`.
Inspect features two different traversal modes, which can be selected via the `use function threshold` button: When the checkbox is turned off, the user can traverse the whole hierarchical mesh and the grid is displayed on the level of the current element. When turned on, the `threshold` of `f_data` and the current `level_of_interest` select the subtree which can be traversed and is displayed. The boundary error estimator is used to draw the blue boundary grid.
The current element is always highlighted in red. Additional debugging-information can be shown in the graphic window or in the shell: Boundary faces can be marked with orange color and global vertex indices can be shown in the graphics window. It is also possible to print some information about the current element and the function on it to `stderr`.

5.6.3 Boundary Display Methods on `HMesh3d`

GRAPE(hmesh, "bnd-isoline-disp")()

GRAPE(mesh, "bnd-isoline-select-disp")()

HMESH3D *hmesh

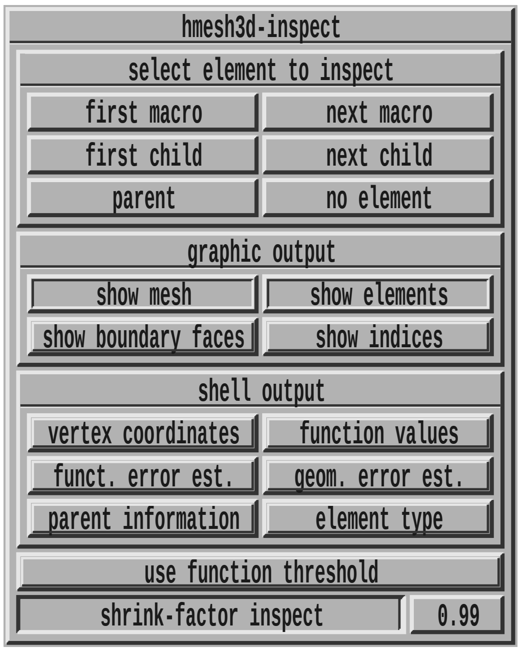


Figure 24: The options layer for `inspect-disp`.

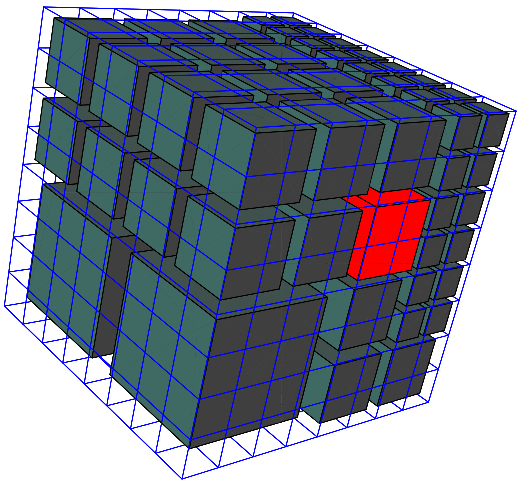


Figure 25: An example of `inspect-disp`.

These methods produce an isoline image of a one dimensional FEM function (or of the absolute value of a vector valued function) at the boundary of the *HMesh3d*-Instance. The method *bnd-isoline-select-disp* displays only those isolines whose values are stored in the array *colorbar−>values*. This method also supports the texture mode of GRAPE making it possible to use a light model together with the use of colors.
The method works especially efficient if the used mesh supports an explicit boundary traversal. To this end there are flags in the *Mesh3d* struct (the super-class of *HMesh3d*) to indicate the currently set traversal mode (*access_mode*) and the supported traversal modes (*access_capability*).

```
typedef struct mesh3d{
  ...
```

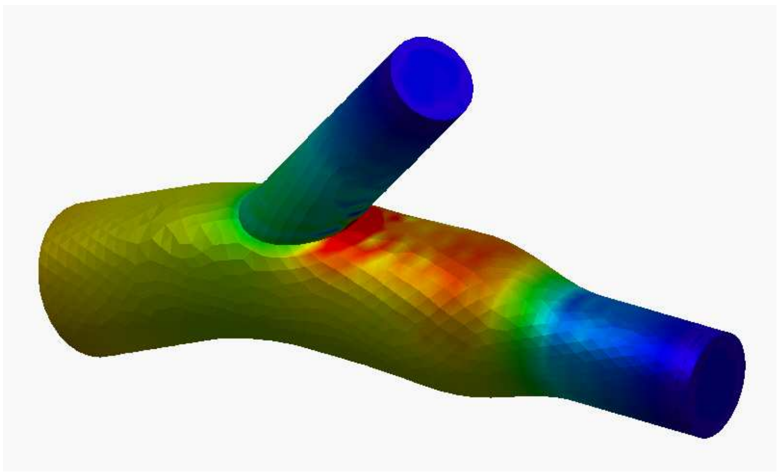


Figure 26: Colorshading of the pressure at the surface of an artery.

```
    MESH_ACCESS_FLAGS  access_mode;
    MESH_ACCESS_FLAGS  access_capability;
  } MESH3D;
```

It is the user's responsibility to set the flag *access_capability* and to program the mesh traversal routines (*first_element*, *next_element*, etc.) so that the set access capabilities are indeed supported. At the moment there are only two possible access flags for different types of mesh traversal:

```
  typedef enum {
    mafNone           = 0,
    mafSorted         = 1<<0,
    mafBoundary       = 1<<1
  } MESH_ACCESS_FLAGS;
```

If the method *bnd-isoline-disp* can set the flag *access_mode* to *mafBoundary* then it will not be checked again if the element is really part of the boundary. Otherwise the method checks each element during the mesh traversal.
Setting or testing the access flags should be done with the methods

```
  GRAPE(hmesh, "get-access-mode")(flags)
    GENMESH *hmesh
    MESH_ACCESS_FLAGS *flags

  GRAPE(hmesh, "set-access-mode")(flags)
    GENMESH *hmesh
    MESH_ACCESS_FLAGS flags
```

GRAPE(hmesh, "bnd-vect-disp")()

HMESH3D *hmesh

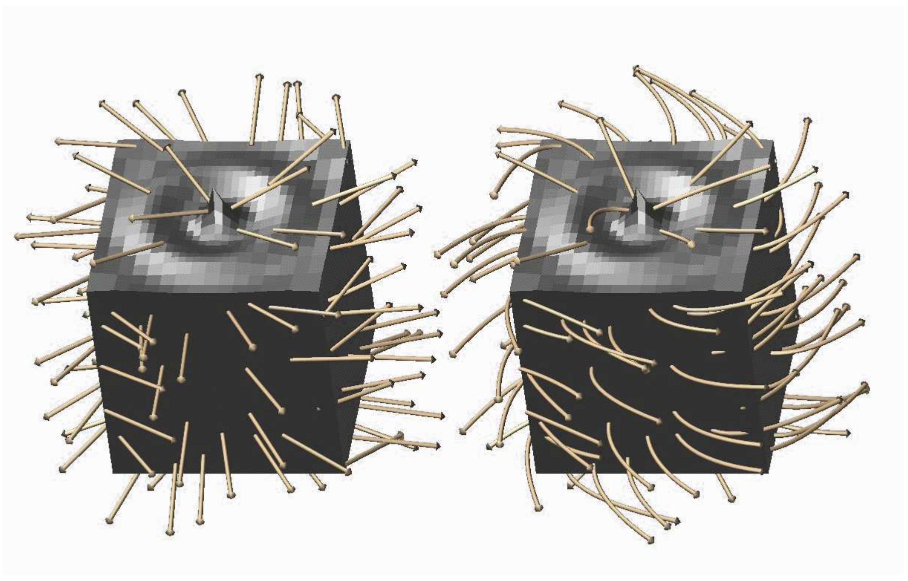
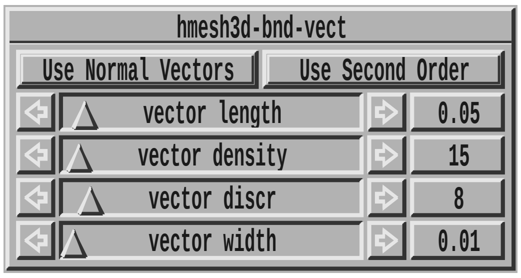


Figure 27: First and second order visualization of a vector field at the boundary of an object.

This method visualizes the data on the boundary of an *HMesh3d* object by drawing random-distributed arrows. If the dimension of the data is less than 3 then the method draws vectors which are orthogonal on the boundary and whose length indicates the absolute value of the data. If the dimension of the data is greater than 3 a warning is printed and the first 3 coordinates of the data are used for the display; it is always possible to force the use of vectors orthogonal to the boundary by pressing the button "Use Normal Vectors". By pressing the button "Use Second Order" curved arrows are drawn giving a second order approximation for the data.



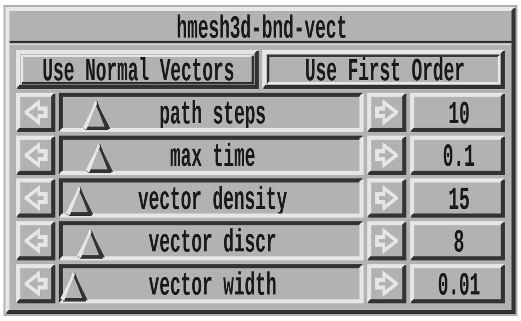
vector length	0.05
vector density	15
vector discr	8
vector width	0.01

Figure 28: Interactive layer of the "bnd-vect-disp" method when the second order mode is not used.

Description of the rulers:

- "vector length" chooses the length of the drawn vectors.
- "vector density" affects the number of arrows drawn.
- "vector discr" determines the discretization level of the drawn arrows (e.g. 1 = line, 2 = flat arrows etc.)
- "vector width" chooses the thickness of the drawn arrows.

New rulers:


hmesh3d-bnd-vect

Use Normal Vectors | Use First Order

path steps 10

max time 0.1

vector density 15

vector discr 8

vector width 0.01

Figure 29: Interactive layer of the "bnd-vect-disp" method when the second order mode is used.

- "path steps" chooses the number of straight segments with which the second order approximation is drawn.
- "max time" determines the end time of the path of a particle moving along the vector field; since the drawn arrow represents a second order approximation to that path, this ruler controls the length of the arrow.

5.6.4 I/O Methods on `HMesh3d`

GRAPE(hmesh, "xdr")(xdr)

XDR *xdrp

This method implements XDR interface of **GRAPE** for the class `HMesh3d`. Of course neither function pointers nor the underlying user data can be handled, but all scalar values are converted, so it is an abstract method. To use the read/write facilities of **GRAPE**, one has to overload this method.

5.6.5 Other Methods on `HMesh3d`

(HMESH3D *)GRAPE(hmesh, "get-bnd-box")(xmin,xmax,ymin,ymax,zmin,zmax)

HMESH3D *hmesh
double *xmin,*xmax,*ymin,*ymax,*zmin,*zmax

Determines the bounding-box of the instance `mesh` and returns it by its extremal coordinates. This method is faster than the `Mesh3d`-version because only macro-elements have to be inspected.

(HMESH3D *)GRAPE(hmesh, "search")(v, c, el)

HMESH3D *hmesh
VEC3 v
double *c
HELEMENT2D **el

This method searches within the `hmesh` for a given point with world-coordinates `v`. If `el` is not NULL local search is performed starting at the element `el`. This means walking up in the

hierarchy until an element is found, which contains `v`, and then walking down to the wanted element. If `el` is NULL at the beginning, hierarchical search is applied by walking over the macro-elements and then walking down the hierarchy.
If successful, the method returns in `el` the element on level `level_of_interest` which contains the given point `v`, and in `c` the local coordinates of this point. The pointer varc has to be allocated with enough memory before calling the method. If the search failed, `el` will be NULL. The search relies on the `HMesh3d`-method `"select_child"`. On one hand this means that this pointer has to be filled in the interface at least with the default-method `"g_hmesh3d_def_select_child"`. On the other hand it implies that implementing this method efficiently is a possibility of speeding up the search.

(HMESH3D *)GRAPE(hmesh, "find-pick-intersec")(v,u,w,el,c)

> **HMESH3D *hmesh**
> **VEC3 v**
> **double u,w**
> **HELEMENT2D **el**
> **double *c**

This method determines for given screen-coordinates `u`,`w` the point on the mesh which is hit by these pick-coordinates. The world coordinates of this point are returned in `v`, the element on level `mesh`→`level_of_interest` which contains this point and its local-coordinates are returned in `el` and `c`. The only difference to the `Mesh3d`-version is the use of improved hierarchical searching.

(HMESH3D *)GRAPE(hmesh, "hmesh-interactive-send")()

> **HMESH3D *hmesh**

This method sets up the project *uif-hm3*, which is described below.

5.6.6 Affiliate Projects

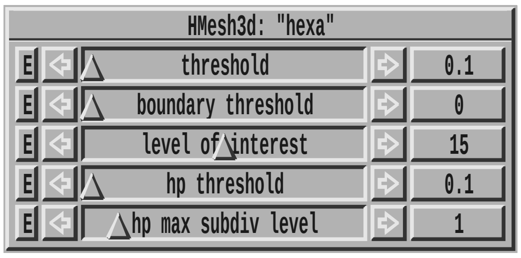


Figure 30: With the HMesh3d interface of the *uif-hm3* project certain parameters of the HMESH3D and F_HDATA structures can be adjusted interactively.

The project *uif-hm3* is the interactive user interfaces for `HMesh3d` instances. It allows editing `max_level_of_interest` and all `threshold` values of the HMESH3D and F_HDATA structure of the current object (cf. Figure 30). One can add them by hand, when needed, or they can be used in the user code with the function `g_project_use(name,batch)` or `g_project_add(name)`. For instance:

```
main ()
{
  ...
  g_project_use ("uif-hm3", g_batch_mode);
  ...
}
```

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