Learning Product Properties with Small Data Sets in Forming Simulations

Rodrigo Iza-Teran, Lukas Morand, Dirk Helm, and Jochen Garcke

Abstract We present a machine learning approach that can learn from simulation data, even if only small data sets are available. Data analysis methods applied to simulation data can be used to get a better understanding of the relations between the parameters and the outcome of the process. But, generally the computational costs of the simulation are an issue and only allow the generation of small sets of numerical data, especially for complex processes. Here, we employ a data representation, which is based on eigenvectors of the Laplace-Beltrami operator for a common underlying geometry, that can already be used for small data sets. To achieve that, the approach assumes an invariance property under certain type of transformations allowing the use of the same operator and consequently the same eigenvectors as orthogonal basis for all deformed shapes. Projections onto this basis enable the comparison of information on the full product geometries, including deformations and field quantities, without the cumbersome procedure of finding and comparing specific features. The basis representation is independent of the available specific simulation result data. We apply the approach to simulation data of a cup drawing process, for which several process parameter variations are performed.

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1 Introduction

Nowadays, product development in industry demands extreme flexibility due to changing customization to meet market demands. Here, numerical process simulations play a deciding role since they can be used to study the effect of multiple parameters on the outcome of the process. However, gaining knowledge out of a set of simulations is a huge challenge. Typically, engineers define specific features that represent the quantities of interest and interpret the data on the basis of these features afterwards. This kind of work is cumbersome and it is not guaranteed that the defined features explain all the relations inside the data set, especially for complex processes. An alternative way to compare simulation data is by applying methods based on shape analysis, so that suitable feature representations are computed based on certain mathematical principles. This shape analysis approach can be employed in case of limited amount of training data and it consists of the following three steps:

1. Compute shape features based on the geometry, e.g., a reference geometry. The features are eigenvectors of the Laplace-Beltrami operator on the shape. The eigenvectors give an orthogonal basis that can be assumed to be the same for all simulations that are using (approximately) the same geometry.
2. A design of experiment is performed, where process parameters are varied inside given ranges in order to obtain a set of simulations. The computed simulation quantities are defined on the mesh and can be projected onto the eigenvector basis in order to obtain a new representation, which is using the coefficients from the corresponding linear combination in the basis. These so-called latent coordinates turn out to be compact in the sense that only few coefficients are enough to describe the coarse variations of the quantities of interest for the simulations.
3. The last step consists of an extended post-processing in the low dimensional latent space and an identification of relevant input parameters correlating with the latent representation.

2 Shape Analysis based Learning Approach

Learning from data of deformation processes is typically based on the assumption that enough samples are available from which one can extract main variation modes and cluster the deformation according to them. An important question is if we can still learn those modes if the available data is scarce. We present a method that partially answers this question using the concept of invariant shape features.

2.1 Invariant Shape Features

In order to describe the generation of the invariant features we consider a Finite Element mesh. The matrix $P \in \mathbb{R}^{N\times 3}$ describes the coordinates of mesh vertices in Euclidean space such that each vertex has the coordinates $\mathbf{p}_i = (x_i, y_i, z_i)$, where N corresponds to the number of nodes. The mesh approximates a continuous surface denoted by $\mathcal{M}$. Functions on it are defined as $f : \mathcal{M} \to \mathbb{R}$ and are continuous functions on $\mathcal{M}$. Evaluating this function at the vertices of the mesh yields the discrete mesh function $f_K : K \to \mathbb{R}$, with K denoting the mesh.

We here consider the Laplace-Beltrami operator on the mesh [1], which is invariant under isometric transformations. These are transformations that preserve geodesic distances on the shape, i.e., distances measured along the surface. This operator is linear, symmetric and positive-semidefinite. Represented as a matrix, its eigendecomposition is given by the eigenvectors $E = [\psi_1, \psi_2, \ldots, \psi_N]$, ordered by the magnitude of the corresponding real eigenvalues, $\lambda_1 < \lambda_2 < \ldots < \lambda_N$. Here, each eigenvector can be seen, in analogy to Fourier analysis, as a frequency component of the mesh function in increasing order.

The normalized eigenvectors E of a symmetric operator form an orthonormal basis. Using the assumption that all transformations of a surface mesh during deformation are isometric, the same basis can be used to represent all deformations. Specifically, a discrete mesh function f_K, given by a vector $\mathbf{f}$, is represented using this basis to obtain a representation in the spectral domain,

$$\hat{\mathbf{f}} = E^\top \mathbf{f}, \tag{1}$$

where the columns of E are eigenvectors $\{\psi_i\}_{i=1}^N$ and $\hat{\mathbf{f}}$ contains corresponding spectral coefficients $\{\alpha_i\}_{i=1}^N$. The spectral coefficients are thus obtained by calculating $\{\alpha_i\}_{i=1}^N = \langle \mathbf{f}, \psi_i \rangle$. The inverse transform reconstructs the mesh function in the spatial domain,

$$\mathbf{f} = E\,\hat{\mathbf{f}}. \tag{2}$$

By considering Euclidean coordinates, $P = \left[\mathbf{f}_x, \mathbf{f}_y, \mathbf{f}_z\right]$, as mesh functions we project the mesh geometry into the spectral domain,

$$\hat{P} = E^\top P, \tag{3}$$

such that each row of $\hat{P}$, $\hat{\mathbf{p}}_i = \left[\alpha_i^x, \alpha_i^y, \alpha_i^z\right], i = 1, \ldots, N$, contains spectral coefficients that can be used to express x-, y-, and z-coordinates as

$$\mathbf{f}_x = \sum_{i=1}^{N} \alpha_i^x \psi_i, \;\; \mathbf{f}_y = \sum_{i=1}^{N} \alpha_i^y \psi_i, \;\; \mathbf{f}_z = \sum_{i=1}^{N} \alpha_i^z \psi_i. \tag{4}$$

An approximation of a coarse representation of the mesh geometry is obtained by using the coefficients corresponding to only the first $M \ll N$ eigenvectors, or suitably selected ones. Functions on the mesh like stress or strain can equally be projected onto the same basis and approximated by few coefficients either. For a detailed description of the approach see [2, 3].

2.2 Post-processing a Simulation Ensemble

Processing all simulation quantities on the mesh using the above explained approach allows us to obtain a low dimensional representation by means of the projection coefficients. A clear overview of all simulations is now possible. The identification of main variation modes as well as clustering turns out to be much simpler than in the original representation.

Identification and Correlation

Given the simulations quantities and the corresponding input parameters that were used, a mapping between input parameters and the outcome of the simulations can be obtained. Applying such methods directly to the 3D data exacerbates most algorithms due to its high dimensionality, which is equal to the number of nodes of the geometry. But, in the new latent representation the identification of relevant input parameters is possible, as well as finding correlations between the obtained representation and the parameters. This is explained in the next section using a metal forming example.

3 Application to Metal Forming

Understanding the outcomes of a forming process affected by multiple input parameters is of high interest for the manufacturing industry. Such knowledge combined with corresponding predictive models can lead to a reduction in scrap and thus to a reduction in costs, or to an increase in product quality, e.g when coupled with process control. However, especially for small and medium-sized enterprises, production processes are often controlled on the basis of empirical knowledge. One reason for this is that usually only limited process data is available, which is furthermore only slightly scattered over the stable process parameter range. Process simulations can provide a remedy, but only to a limited extent, due to limitations of computational power. In the following, we analyze simulation data of a cup drawing process by using the above described machine learning method. For a comparison with a knowledge-based approach see [4].

3.1 Cup drawing simulation

Cup drawing is a typical sheet metal forming process that can be used as a testing procedure for the formability of sheet metal and has been investigated intensively, see for example [5, 6, 7]. In a cup drawing process, a punch presses a round sheet metal into a die while a blank holder holds the sheet on the edge of the die in order to avoid wrinkles. The experimental setup of the cup drawing process that is modeled in the present work is described in [8]. In order to use the Finite Element Method, a quarter of the cup is modeled obeying symmetry conditions and is discretized via shell elements. The material is described using an elasto-plastic material model. The plastic behavior is defined by an exponential hardening model [9]:

$$\sigma = \sigma_0 + k\varepsilon_{\text{pl}}^n, \tag{5}$$

in combination with the Hill'48 anisotropic yield criterion [10]. Furthermore, to model initiating damage, a criterion is used that is based on forming limit curves (FLDCRT). These curves are approximated using the Keeler-Brazier equation [11]:

$$\begin{aligned}
\varepsilon_1 &= \text{FLD}_0^{\text{True}} - \varepsilon_2, \quad \varepsilon_2 < 0 \\
\varepsilon_1 &= \ln(0.6(\exp(\varepsilon_2) - 1) + \exp(\text{FLD}_0^{\text{True}})), \quad \varepsilon_2 > 0 \\
\text{FLD}_0^{\text{True}} &= ln(1 + (0.233 + 0.413t)\frac{n}{t}),
\end{aligned} \tag{6}$$

where n is the strain hardening coefficient and t the thickness of the sheet metal. The velocity of the punch is constant during the forming process. Individual snap-shots of the process simulation are shown in Figure 1.

Using the FEM software Abaqus/Explicit, a data base for the cup drawing process is generated. In our scenario the influence of process parameters and typical material

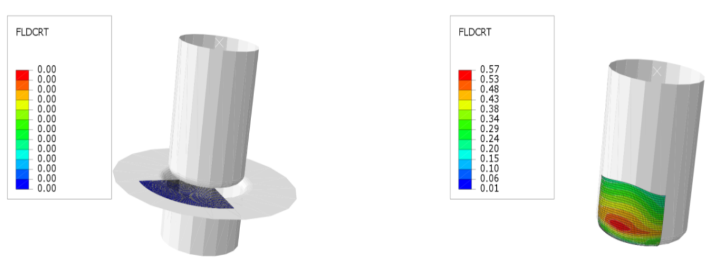

Fig. 1 Cup drawing simulation. Left: The initial state of the simulation. One can see the round sheet, the punch and the die. Right: As the punch presses the sheet into the die, a round cup is formed. One can see the drawn sheet and the punch. For a better visualization the blank holder device is hidden in both pictures. This is the counterpart of the die, which presses the sheet metal onto the upper edge of the die. The colour coding refers to the degree of deformation in terms of the forming limit diagram.

Table 1 Varied parameters, their ranges and units.

parameter	F_{BH}	μ	t	σ_0	k	n	R_{00}	R_{45}	R_{90}
min	22000	0.07	0.9	160	1	0.15	1.0	1.0	1.0
max	133500	0.22	1.1	280	250	0.25	2.3	2.3	2.3
unit	N	-	mm	MPa	MPa	-	-	-	-

properties for DC04 steels on the deformed shape is investigated. These parameters are: blank holder force F_{BH}, friction between tools and sheet metal μ, sheet metal thickness t, parameters of the hardening model $\{\sigma_0, k, n\}$, and the Lankford coefficients at 0, 45 and 90 degree to rolling direction that serve as inputs for the Hill'48 yield criterion $\{R_{00}, R_{45}, R_{90}\}$. The parameter ranges are defined according to values for process parameters found in [7] and [8] and for DC04 sheet metal properties defined in the DIN norms [12] and [13], plus that we assume fluctuations from these values of about 10%. The varied parameters, their ranges and units are listed in Table 1. In total, the data base consists of 9998 data points with randomly drawn parameter variations. The only restriction is put on the coefficient R_{45}, which is always lower than or equal to the minimum of R_{00} and R_{90}.

3.2 Application of the Method based on Shape Analysis

The obtained simulation data is transformed into a lower dimensional representation following the approach from Section 2. The representation is computed by the projection of mesh functions, such as deformations or field quantities, onto the eigenvectors obtained from the shape. The 3D deformations are given as three functions on the mesh: x, y and z, therefore we get a total of $N \times 3$ coefficients, where N is the number of nodes. But, only the first few coefficients are big and correspond to coarse changes in the mesh function, in this case the deformation. In order to select the required coefficients, a criterion is needed. For this work we use the variance with respect to all given projections and we choose three coefficients to be able to represent it using three latent variables. Figure 2 shows this 3D representation. A first use of it consists in identifying the shapes that are deformed excessively. This is easily done by selecting the points along the elongated curved cluster from the middle to the right hand side.

Now, we would like to analyze the influence of the input variables for the selected shapes with stronger deformations. As seen in Figure 3, these shapes are deformed depending on the thickness specified as input parameter. Notice that the latent coordinates correlate to the thickness, which at the same time implies that this input variable is a dominant one in relation to all others.

A further analysis is done on the mesh function FLDCRT. As before, we project it onto the same eigenbasis and choose a few coefficients based on their variance, allowing us to obtain another latent representation. In Figure 4 a correlation between the latent coordinates and the hardening exponent is clearly visible and it seems to

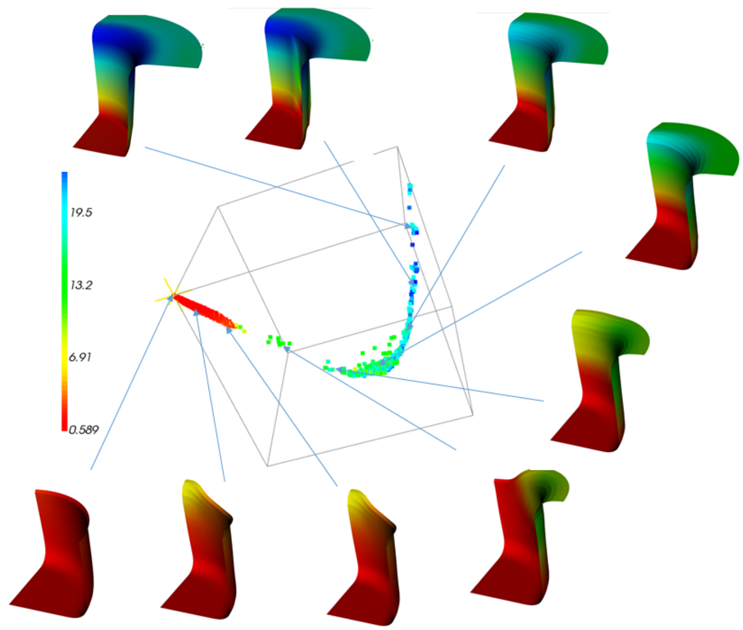

Fig. 2 Representation of a simulation ensemble in a 3D space of the projection coefficients. Each point represents a shape at the end of the forming process. The points are colored by a simulation parameter and the color coding of the shapes correspond to the deformation differences to a reference simulation, where red indicates zero difference. Two groups are of interest, one "straight" cluster on the left hand side and an elongated curved cluster from the middle to the right hand side.

be stratified on some regions. A second relevant variable has been identified and not only that, its influence on the function FLDCRT can be clearly observed by following the points along one of the clusters in Figure 4. Such an investigation on shapes with strong deformations, which in metal forming actually are not aimed for, allows us to analyse the influence of the input parameters and identify the most dominant ones.

As a final task we concentrate on the cluster containing cups that did not got stuck in the die. The interesting question is whether we can differentiate within those shapes and also if we can find a dependence to some input variable. First, we use the latent points in Figure 2 and select only those points in the smaller region on the left colored with red points. These points represent shapes that have much less deformation. As each point in the latent space represents a simulation, we can further take only those simulations for the analysis. Now, using the same basis as was used before for analysis in this work, we choose the projection coefficients in the z direction and ignore the ones in x and y directions. Next, we choose the first three with high variance. Figure 5 shows the obtained latent points together with a few chosen deformation shapes. In the lower region of the 3D representation, shapes with smaller lateral deformation in the z direction can be observed. Further, in the upper part, the shapes are clustered with deformations on only one side of the cup

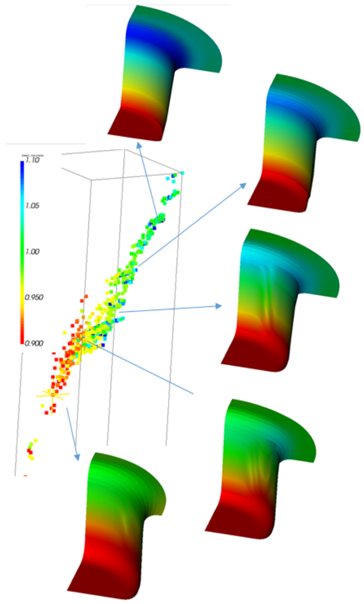

Fig. 3 Latent representation of the 3D deformations (mesh functions x, y and z) showing a relation to the thickness (color of the points). The color on the shape shows the differences to a reference shape.

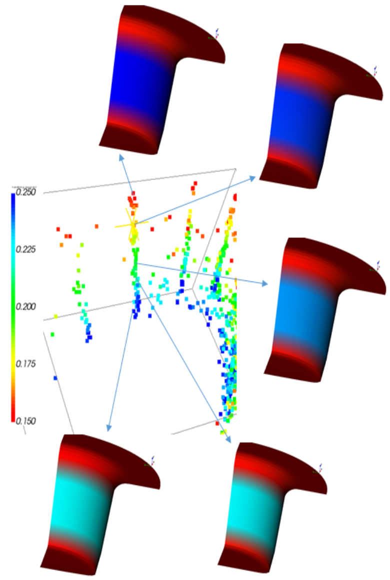

Fig. 4 Latent representation of the mesh function FLDCRT showing a dependence with respect to hardening coefficient (color of the points). The color of the shape represents the function FLDCRT itself.

(earing). Notice that the latent representation shows a certain symmetry where points on the upper left correspond to shapes with deformations higher on their right hand side and vice-versa. The color of the points corresponds to the input variable R_{00} and according to the distribution on the 3D point cloud, lower values of this variable can be associated with more uniform shapes. This is a realistic observation and proves the applicability of the method, as R-values equal to one correspond to isotropic behavior of the sheet metal, which is known to not cause earing.

These are but a few examples of the capabilities of the proposed simulation data analysis method. We notice that the regions with correlations as found in this use case are well suited for the construction of surrogate models between the identified relevant parameters and the corresponding latent points. This will be the subject of further investigations.

4 Summary and Conclusion

A latent representation based on shape analysis has been presented. The approach constructs a unique orthogonal basis, valid for all simulation quantities on a finite

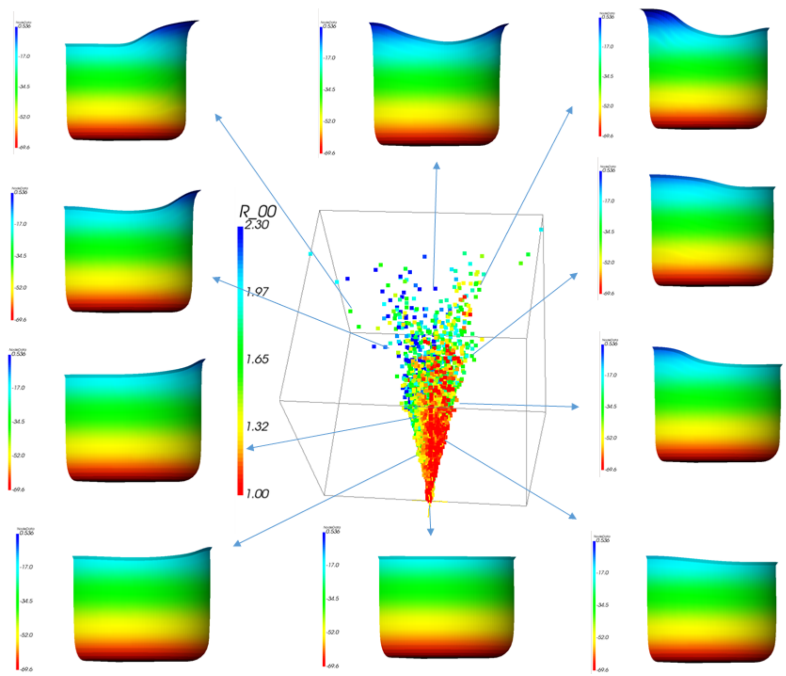

Fig. 5 Representation of a cluster where each point represent a shape at the end of the forming process. The points are colored by the input parameter R_{00} and the color coding of the shapes correspond to the deformation differences in direction z to a reference simulation, where red indicates zero difference.

element mesh and can be used with a limited amount of available data. The workflow has been described and exemplary applied to a cup drawing process under the influence of several input parameters. An overview of the simulation data has been obtained by means of a low dimensional latent representation in three dimensional space. Relevant parameters have also been identified in the latent space. The analysis of the effects of design parameters on simulation quantities has shown to be feasible with the proposed procedure.

Note, that a comparison with a baseline method such as Principal Component Analysis (PCA) is necessary. First results show that PCA is also able to represent the variability of the analyzed simulation data in very few components. But, clustering using the PCA representation describes only the variance of the training data. Whether or not those projections correlate with the input parameters is not clear. In contrast to PCA, our proposed approach has a high flexibility in its application. The same shape-derived basis can be used to analyse the variation of the shapes, corresponding to geometric features, or to analyse the variation of mesh functions defined on the geometry. With the presented approach one is able to identify correlations between relevant input parameters and the projection coefficients, i.e., the latent representation. Note that having few training data for analysis using the proposed approach will just result in clusters which are more sparse. Their shapes and distributions do not change, allowing overall the same investigations and conclusion, e.g., regarding input parameter dependence.

More research is indeed warranted to fully evaluate the capabilities of the method and especially to automatically detect which mesh function shows interesting latent representations with respect to given input parameters. Once correlations between input variables and the corresponding latent representation are found, the next task is to construct surrogate models to learn a mapping between them.

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