Numerical verification of the bond-based Peryidynamic softening model against classical theory

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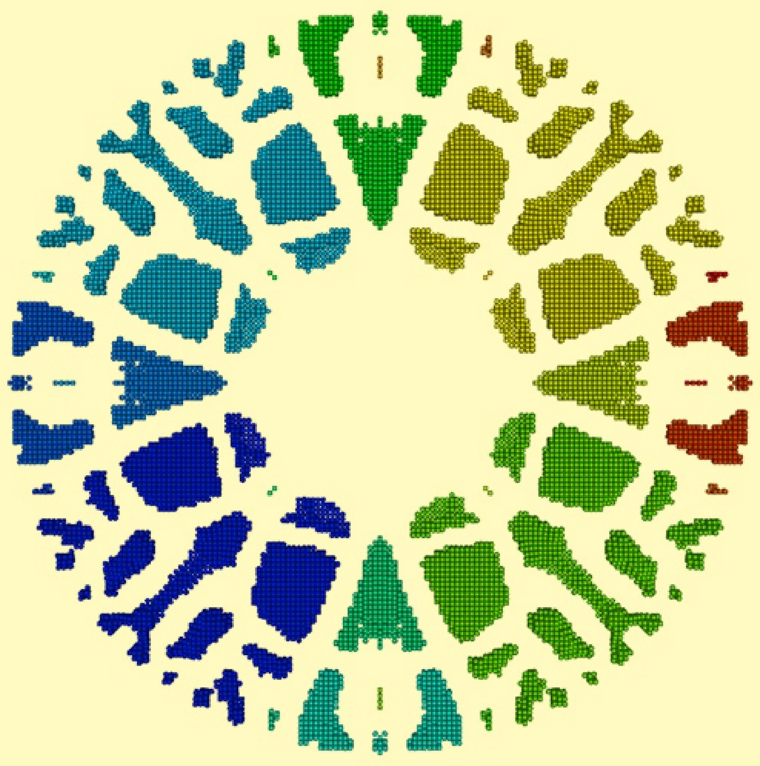
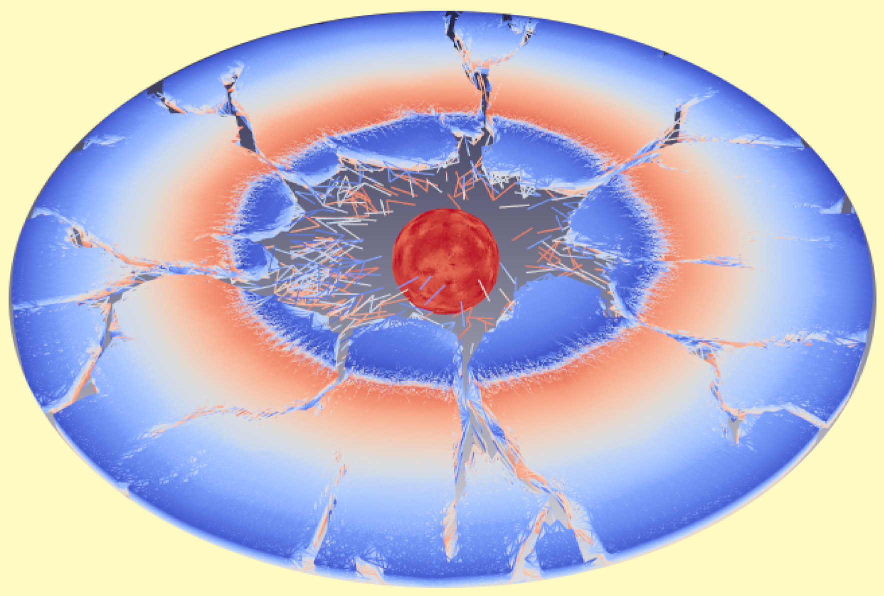
Abstract

We introduce the bond-based Peridynamic Softening material model with respect to small strain deformation. A specialty of this model is, that the the material parameter are independent of the size of the neighborhood. This model is derived from classical theory. Thus, the model parameters can be obtained from material parameters. We verified the discrete version of the Softening model with the reproduction of the Poisson's ratio and the Young's modulus and showed the conversion of the Peridynamic energy density.

Motivation



Fragmentation



Damage and fracture zones

Bond-based Peridynamics

PD equation of motion (Continuum)

$$\rho(X)A(t,X) = \int\limits_{B_\delta(X)} f\left(t, x(t,X') - x(t,X), X' - X\right) dX'$$

Discrete PD equation of motion (EMU)

$$\rho(X_i)A(t,X_i) = \sum_{X_j \in B_\delta(X_i)} f\left(t, x(t,X_j) - x(t,X_i), X_j - X_i\right) \frac{X_j - X_i}{\|X_j - X_i\|} dX_j$$

Silling, S. & Askari, E: A meshfree method based on the peridynamic model of solid mechanics, Computer & Structures, 2005, 83, 1526–1535

Softening Model

Discrete bond-based softening model

$$\rho(X_i)A(t,X_i) = \frac{1}{V_d} \sum_{X_j \in B_\delta(X_i)} f\left(t, x(t,X_j) - x(t,X_i), X_j - X_i\right) \frac{X_j - X_i}{\|X_j - X_i\|} dX_j$$

Pair-wise force function $f : [0,T] \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

$$f(t, x(t,X_j) - x(t,X_i), X_j - X_i) := \frac{2}{\delta} \alpha\beta s(t, x(t,X_j) - x(t,X_i), X_j - X_i)$$
$$\exp\left(-\alpha \|X_j - X_i\| s^2(t, x(t,X_j) - x(t,X_i), X_j - X_i)\right)$$

Small strain stretch $s : [0,T] \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

$$s(t, X_i, X_j) := \frac{(X_{j_1} - X_{i_1})^2}{\|X_j - X_i\|^2}$$

R. Lipton, Dynamic Brittle Fracture as a Small Horizon Limit of Peridynamics, Journal of Elasticity, 2014, Volume 117, Issue 1, pp 21-50.

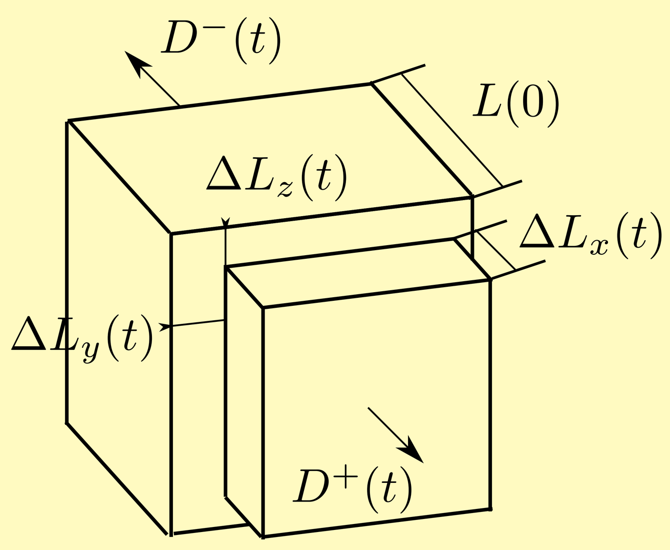
Material properties

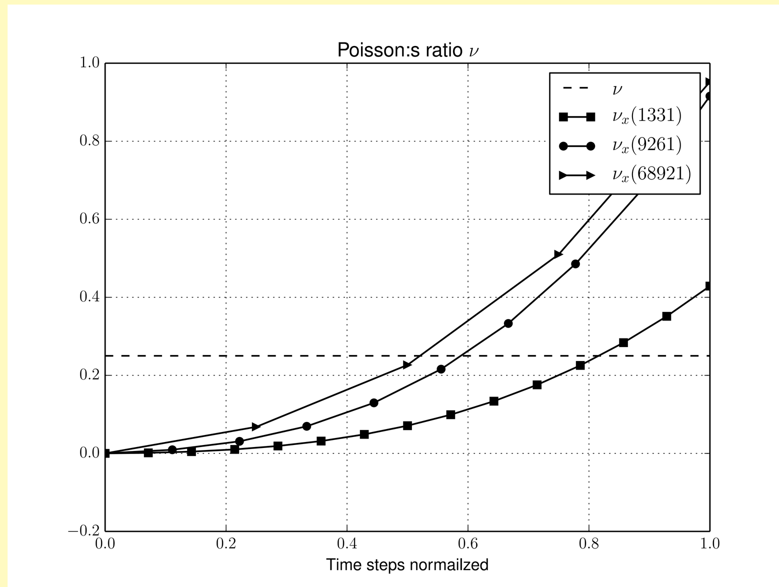
	Model	Energy equivalence
Fracture Toughness	$G := \frac{6}{16}\beta$	$\beta(K, K_{\text{Ic}}) := \frac{16K_{\text{Ic}}^2}{9K}$
First Lamé parameter	$\lambda := \frac{1}{20}\alpha\beta$	$\alpha(K,\beta) := 12\frac{K}{\beta}$

All parameters are independent on the horizon δ!

Verification against classical theory

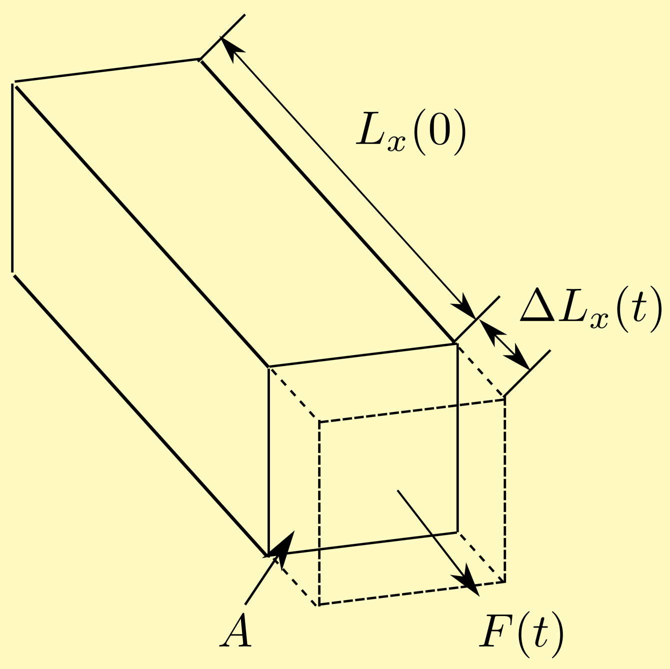
- Recovering the Poisson's ratio $\nu = 0.25$

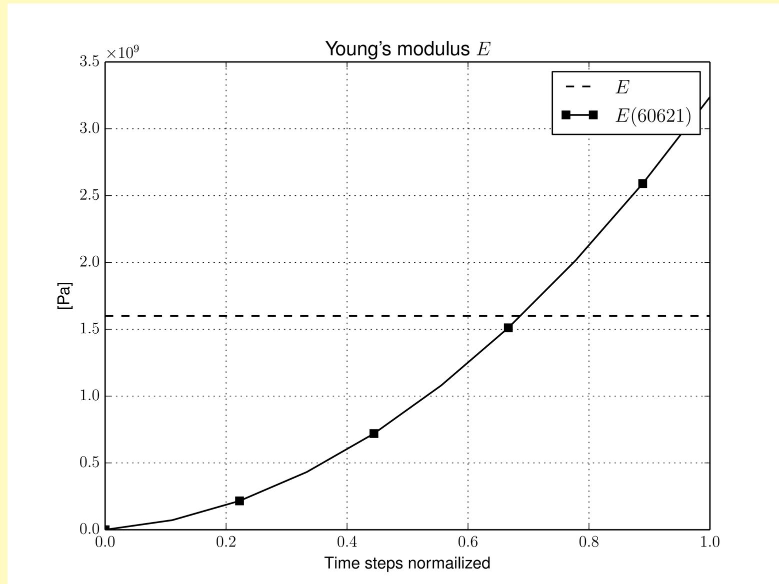




$$\nu_x(t) = \frac{\Delta L_y(t)}{\Delta L_x(t)}$$

- Recovering the Young's modulus $E = 1.6$GPa





$$E(t) = \frac{F(t)L_x(0)}{A\Delta L_x(t)}$$

Verification against model

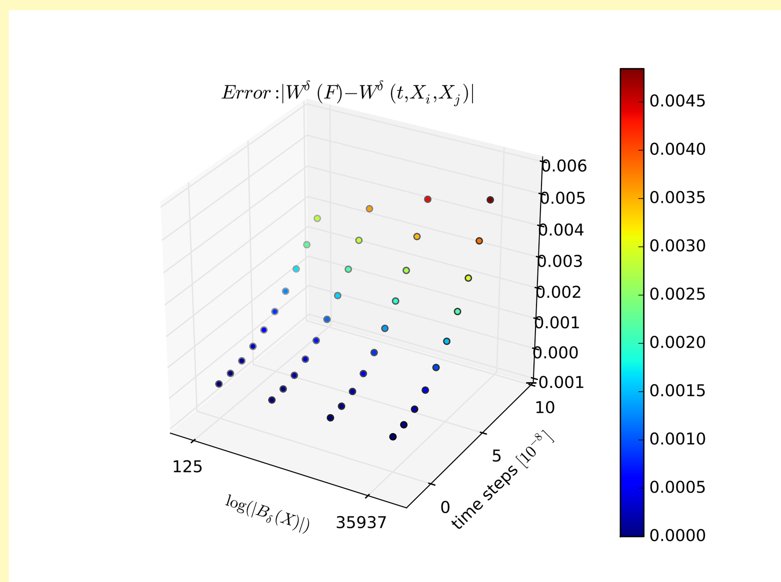
Peridynamic energy $W_M : \mathbb{R}^{3\times3} \to \mathbb{R}$ obtained by the model

$$W_M(F) := 2\mu|F|^2 + \lambda tr(F)^2, \quad \text{with } F := \begin{pmatrix} s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Discrete Peridynamic energy $W : [0,T] \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

$$W^\delta(t, X_i, X_j) := \frac{1}{\delta V_3} \alpha\beta s(t, X_i, X_j)$$

Comparison of the discrete and model Peridynamic energy density



Conclusion & Outlook

Conclusion

- Material parameters are independent on δ
- Reproduce Young's modulus E and Poisson's ratio ν up to 10%
- Recovering of the energy $W^\delta(t, X_j, X_i)$ up to 2%

Outlook

- Optimize simulations for Young's modulus E and Poisson's ratio ν
- Compare the energy around the fracture with the Griffith fracture energy