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Removing rotational modes

Suppose we compute a velocity field v_0 and would like to remove the rotational component. Our goal is to compute a correction v such that v is a rigid body rotation and $v_0 - v$ is non-rotational.

In physics, velocity is not a conserved quantity, but the momentum is. Assuming a density ρ we compute the angular momentum

$$M_0 = \int_\Omega r \times \rho v_0 \, dx . \qquad (1)$$

r is the vector from the origin. For the correction we make the ansatz

$$v = \omega \times r \qquad (2)$$

with unknown rotational axis and speed ω:

$$M = \int_\Omega r \times \rho (\omega \times r) dx = \int_\Omega \rho \left(\omega r^2 - r (r \omega) \right) dx$$

$$= I \omega \qquad (3)$$

with moment of inertia

$$I = \int_\Omega \rho \left(r^2 1 - r r^T \right) dx . \qquad (4)$$

We can then compute the correction by

$$\omega = I^{-1} M_0 . \qquad (5)$$

For the special case of a solid sphere of constant density and radius R the tensor I becomes diagonal,

$$I_R = \rho \int_{S_R} \frac{2}{3} r^2 \, dx = \frac{2}{3} \rho \int_0^R r^2 \, 4\pi r^2 \, dr$$

$$= \frac{8}{15} \pi \rho R^5 = \frac{2}{5} m R^2 \qquad (6)$$

(m is the mass of the sphere). We cancel the density in (1) for

$$\omega = \left(\frac{8}{15} \pi R^5 \right)^{-1} \int_\Omega r \times v \, dx . \qquad (7)$$

For a hollow sphere the computation needs to be modified. This is published in

Zhong, McNamara, Tan et. al., "A benchmark study on mantle convection in a 3D spherical shell using CitcomS", Gcubed 9, 2008, pages Q 10017.

Effectively, in (7) we replace R^5 with $R_1^5 - R_0^5$ for the hollow sphere $\Omega(R_1, R_0)$.